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Logarithmic Comparison with Smooth Boundary Divisor in Mixed Hodge Modules

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1. Introduction

The main goal of this paper is to study certain filtered log-$\mathscr{D}$-modules that underlie the (dual) localization of Saito's mixed Hodge modules along a smooth hypersurface (or more generally, it admits a multi-indexed Kashiwara–Malgrange filtration with respect to a normal crossing divisor as defined in Section 4) and show that they also behave well under the direct image functor and the duality functor in the derived category of filtered log-$\mathscr{D}$-modules. We will apply the results of this paper to prove a natural and substantial generalization of the result of Popa and Schnell [PS14] in the log setting. This generalization will appear in [Wei17]. Another application for the results in this paper is to simplify the proof of Viehweg's hyperbolicity for families of smooth varieties of general type in [PS17] and to generalize the result to the case of log-smooth families. Some other potential applications can also be expected in studying birational geometry of families of log-pairs over a log-smooth variety, for example, subadditivity of log-Kodaira dimensions [Fuj17].

We will consider right (log-)$\mathscr{D}$-modules in this paper, if not otherwise specified. The mixed Hodge modules that we are discussing here are all assumed to be algebraic. In particular, they are extendable [Sai90, Section 4]. We mainly follow the notations that appeared in [SS16]. In particular, $\widetilde{\mathscr{D}}$ is the Rees algebra induced by the filtration F on $\mathscr{D}$ given by the degree of the differential operators. We denote by $\widetilde{\mathbb{C}}_X$ and $\widetilde{\mathscr{O}}_X$ the corresponding Rees algebra induced by the trivial filtration. See Section 2 for more details. We say a strict right $\widetilde{\mathscr{D}}$-module is a mixed Hodge module if it underlies a mixed Hodge module in the sense of Saito [Sai88; Sai90], forgetting the weight filtration. All algebraic varieties that we work with in this paper are smooth and over the complex number field $\mathbb{C}$.

Fix a normal crossing divisor D on X. Let $\mathscr{D}_{(X,D)}$ be the ring of log-differential operators of the log-smooth pair (X, D), which is canonically a sub-ring of $\mathscr{D}_X$. Similarly, denote by $\widetilde{\mathscr{D}}_{(X,D)}$ the Rees algebra, induced by the filtration F given by the degree of the differential operators. The two $\widetilde{\mathscr{D}}_{(X,D)}$-modules that we are most interested in in this paper are the following.

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