Michigan Math. J. 66 (2017), 481–498

Lefschetz Fibrations on Knot Surgery 4-Manifolds Via Stallings Twist

Jongil Park & Ki-Heon Yun

Abstract. In this paper, we construct a family of simply connected minimal symplectic 4-manifolds that admit arbitrarily many nonisomorphic Lefschetz fibration structures with the same genus fiber. We obtain these families by performing knot surgery on an elliptic surface $E(2)$ using connected sums of n copies of fibered knots, which in turn are obtained by Stallings twist from the square knot. Thus, all of these 4-manifolds are homotopy $E(2)$ surfaces. We show that they admit 2^n mutually nonisomorphic Lefschetz fibration structures of fiber genus $(4n+1)$ by comparing their monodromy groups that are induced from the corresponding monodromy factorizations.

1. Introduction

Since it was known that any closed symplectic 4-manifold admits a Lefschetz pencil [Don99] and that a Lefschetz fibration structure can be obtained from a Lefschetz pencil by blowing-up the base loci, the study of Lefschetz fibrations has become an important research theme for understanding symplectic 4-manifolds topologically. In fact, Lefschetz pencils and Lefschetz fibrations have long been studied extensively by algebraic geometers and topologists in the complex category, and these notions can be extended to the symplectic category. It is also well known that an isomorphism class of Lefschetz fibrations over S^2 is completely characterized by *monodromy factorization*, an ordered sequence of right-handed Dehn twists whose product becomes the identity in the surface mapping class group corresponding to the generic fiber, up to Hurwitz equivalence and global conjugation equivalence. Note that the Hurwitz equivalence problem of monodromy factorizations provides a very interesting but challenging question in topology. For example, one aim for researchers in this field is to answer the following questions:

> Is the Hurwitz problem for mapping class group factorizations decidable? Do interesting criteria exist that can be used to conclude that two given factorizations are equivalent, or inequivalent, up to Hurwitz moves and global conjugation? [Aur06]

On the other hand, since the introduction of gauge theory, in particular Seiberg–Witten theory, topologists and geometers working on 4-manifolds have developed various techniques, and many fruitful and remarkable results have been obtained regarding the topology of 4-manifolds over the last 30 years. Among these, a knot surgery technique introduced by R. Fintushel and R. Stern turned

Received July 8, 2015. Revision received February 23, 2017.