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Difference Nevanlinna Theories with Vanishing and Infinite Periods

Yik-Man Chiang & Xu-Dan Luo

Dedicated to the memory of J. Milne Anderson.

Abstract. By extending the idea of a difference operator with a fixed step to a varying-step difference operator, we establish a difference Nevanlinna theory for meromorphic functions with steps tending to zero (vanishing period) and a difference Nevanlinna theory for finite-order meromorphic functions with steps tending to infinity (infinite period). We can recover the classical little Picard theorem from the vanishing period theory, but we require additional finite-order growth restriction for meromorphic functions from the infinite period theory. Then we give some applications of our theories to exhibit connections between discrete equations and and their continuous analogues.

1. Introduction

Halburd and Korhonen [9] established a new Picard-type theorem and Picard values with respect to difference operator $\Delta f(z) = f(z+1) - f(z)$ for finite-order meromorphic functions defined on $\mathbb{C}$ versus the classical Picard theorem and Picard values. More specifically, their theory allows them to show that if there are three points a_j, $j = 1, 2, 3$, in $\widehat{\mathbb{C}}$ such that each preimage $f^{-1}(a_j)$ is an infinite sequence consisting of points lying on a straight line on which any two consecutive points differ by a fixed difference c (but is otherwise arbitrary), then the function must be a periodic function with period c. This result can be considered as a discrete version of the classical little Picard theorem for finite-order meromorphic functions. A crucial tool of their theory, which follows from their difference-type Nevanlinna theory for finite-order meromorphic functions, is the difference logarithmic derivative lemma [8] (see also [3]), that is,

$$m\left(r, \frac{f(z+c)}{f(z)}\right) = o(T(r, f)),$$

where c is a fixed nonzero constant. Instead of a fixed c, we define $g(z, c) := f(z+c)$, where $(z, c) \in \mathbb{C}^2$. Then $f(z+c)$ is a meromorphic function in $\mathbb{C}^2$.

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