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Infinite Groups Acting Faithfully on the Outer Automorphism Group of a Right-Angled Artin Group

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Abstract. We construct the first known examples of infinite subgroups of the outer automorphism group of $\mathrm{Out}(A_\Gamma)$, for certain right-angled Artin groups A_Γ. This is achieved by introducing a new class of graphs, called *focused graphs,* whose properties allow us to exhibit (infinite) projective linear groups as subgroups of $\mathrm{Out}(\mathrm{Out}(A_\Gamma))$. This demonstrates a marked departure from the known behavior of $\mathrm{Out}(\mathrm{Out}(A_\Gamma))$ when A_Γ is free or free abelian since in these cases $\mathrm{Out}(\mathrm{Out}(A_\Gamma))$ has order at most 4. We also disprove a previous conjecture of the second author, producing new examples of finite-order members of certain $\mathrm{Out}(\mathrm{Aut}(A_\Gamma))$.

1. Introduction

Right-angled Artin groups, or *RAAGs,* comprise a class of groups that generalize free groups and free Abelian groups. Every finite simplicial graph Γ with vertex set V defines a RAAG A_Γ in the following way. The generating set of A_Γ is in bijection with the vertices of Γ, and the only relations are that two generators commute if their corresponding vertices share an edge in Γ. Thus, if Γ has no edges, then A_Γ is just the free group F_V, whereas if Γ is a complete graph, then A_Γ is the free Abelian group $\mathbb{Z}\langle V\rangle$.

In this paper, we consider the automorphism and outer automorphism groups of general RAAGs in comparison with those of free groups and free Abelian groups. More specifically, we investigate $\mathrm{Out}(\mathrm{Out}(A_\Gamma))$ and $\mathrm{Out}(\mathrm{Aut}(A_\Gamma))$. These groups provide a measure of the algebraic rigidity of $\mathrm{Out}(A_\Gamma)$ and $\mathrm{Aut}(A_\Gamma)$, respectively, and their study fits into a more general program of investigating rigidity of groups throughout geometric group theory.

The main goal of this paper is to show that there exist infinitely many graphs Γ for which $\mathrm{Out}(\mathrm{Out}(A_\Gamma))$ is infinite. We achieve this by introducing a new class of graphs, which we call *focused graphs.* A graph Γ is said to be *focused* if it has a distinguished vertex c with the following two properties: (i) c is the unique vertex of Γ that may dominate a vertex other than itself, and (ii) c is the only vertex whose star disconnects Γ. Focused graphs are the key construction that allows us to prove our following main theorem.

Theorem A. *For each $n \geq 2$, there exist infinitely many focused graphs Γ such that* $\mathrm{Out}(\mathrm{Out}(A_\Gamma))$ *contains* $\mathrm{PGL}_n(\mathbb{Z})$.

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