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Jet Schemes and Generating Sequences of Divisorial Valuations in Dimension Two

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Abstract. Using the theory of jet schemes, we give a new approach to the description of a minimal generating sequence of a divisorial valuations on $\mathbf{A}^2$. For this purpose, we show how to recover the approximate roots of an analytically irreducible plane curve from the equations of its jet schemes. As an application, for a given divisorial valuation v centered at the origin of $\mathbf{A}^2$, we construct an algebraic embedding $\mathbf{A}^2 \hookrightarrow \mathbf{A}^N$, $N \geq 2$, such that v is the trace of a monomial valuation on $\mathbf{A}^N$. We explain how results in this direction give a constructive approach to a conjecture of Teissier on resolution of singularities by one toric morphism.

1. Introduction

Let $X = \mathbf{A}^d = \operatorname{Spec} R$, where $R = \mathbf{K}[x_1, \dots, x_d]$ is a polynomial ring over an algebraically closed field $\mathbf{K}$. The arc space of X, which we denote by X_∞, is the scheme whose $\mathbf{K}$-rational points are

$$X_\infty(\mathbf{K}) = \operatorname{Hom}_{\mathbf{K}}(\operatorname{Spec} \mathbf{K}[[t]], X).$$

We have a natural truncation morphism $X_\infty \longrightarrow X$, which we denote by Ψ_0. For $p \in \mathbf{N}$ and the subvariety $Y = V(I) \subset X$ defined by an ideal I, we consider the subset of arcs in X_∞ that have an order of contact p with Y, that is,

$$\operatorname{Cont}^p(Y) = \{\gamma \in X_\infty \mid \operatorname{ord}_t \gamma^*(I) = p\},$$

where $\gamma^* : R \longrightarrow \mathbf{K}[[t]]$ is the $\mathbf{K}$-algebra homomorphism associated with γ, and

$$\operatorname{ord}_t \gamma^*(I) = \min_{h \in I}\{\operatorname{ord}_t \gamma^*(h)\}.$$

With an irreducible component $\mathbb{W}$ of $\operatorname{Cont}^p(Y)$, which is contained in the fiber $\Psi_0^{-1}(0)$ above the origin, we associate a valuation $v_{\mathbb{W}} : R \longrightarrow \mathbf{N}$ as follows:

$$v_{\mathbb{W}}(h) = \min_{\gamma \in \mathbb{W}}\{\operatorname{ord}_t \gamma^*(h)\} \quad \text{for } h \in R.$$

It follows from [ELM] (see also [dFEI; Re], Prop. 3.7(vii)) that $v_{\mathbb{W}}$ is a divisorial valuation centered at the origin $0 \in X$ and that all divisorial valuations centered at $0 \in X$ can be obtained in this way.

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