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A Surface with $q = 2$ and Canonical Map of Degree 16

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Abstract. We construct a surface with irregularity $q = 2$, geometric genus $p_g = 3$, self-intersection of the canonical divisor $K^2 = 16$, and canonical map of degree 16.

1. Introduction

Let S be a smooth minimal surface of general type. Denote by $\phi : S \dashrightarrow \mathbb{P}^{p_g-1}$ the canonical map, and let $d := \deg(\phi)$. The following Beauville's result is well known.

Theorem 1 [Be]. *If the canonical image* $\Sigma := \phi(S)$ *is a surface, then either*:

(i) $p_g(\Sigma) = 0$, *or*

(ii) Σ *is a canonical surface* (*in particular*, $p_g(\Sigma) = p_g(S)$).

Moreover, in case (i) $d \le 36$, *and in case* (ii) $d \le 9$.

Beauville has also constructed families of examples with $\chi(\mathcal{O}_S)$ arbitrarily large for $d = 2, 4, 6, 8$ and $p_g(\Sigma) = 0$. Despite being a classical problem, for $d > 8$ the number of known examples drops drastically: only Tan's example [Ta, §5] with $d = 9$, the author's [Ri] example with $d = 12$, and Persson's example [Pe] with $d = 16$ are known. There is a recent preprint of Sai-Kee Yeung [Ye] claiming that the case $d = 36$ does occur. Du and Gao [DuGa] show that if the canonical map is an Abelian cover of $\mathbb{P}^2$, then the examples mentioned with $d = 9$ and $d = 16$ are the only possibilities for $d > 8$. These surfaces are regular, so for irregular surfaces, all known examples satisfy $d \le 8$. We get from Beauville's proof that lower bounds hold for irregular surfaces. In particular,

$$q = 2 \quad \Longrightarrow \quad d \le 18.$$

In this note, we construct an example with $q = 2$ and $d = 16$. The idea of the construction is the following. We start with a double plane with geometric genus $p_g = 3$, irregularity $q = 0$, self-intersection of the canonical divisor $K^2 = 2$, and singular set the union of 10 points of type A_1 (nodes) and 8 points of type A_3 (standard notation, the resolution of a singularity of type A_n is a chain of (-2)-curves $C_1, \dots, C_n$ such that $C_iC_{i+1} = 1$ and $C_iC_j = 0$ for $j \neq i \pm 1$). Then we take a double covering ramified over the points of type A_3 and obtain a surface with $p_g = 3$, $q = 0$ and $K^2 = 4$ with 28 nodes. A double covering ramified over 16 of these 28 nodes gives a surface with $p_g = 3$, $q = 0$ and $K^2 = 8$ with 24 nodes (which is a $\mathbb{Z}_2^3$-covering of $\mathbb{P}^2$). Finally, there is a double covering ramified

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