

# Compactification of Certain Clifford–Klein Forms of Reductive Homogeneous Spaces

FRANÇOIS GUÉRITAUD, OLIVIER GUICHARD,  
FANNY KASSEL, & ANNA WIENHARD

ABSTRACT. We describe smooth compactifications of certain families of reductive homogeneous spaces such as group manifolds for classical Lie groups, or pseudo-Riemannian analogues of real hyperbolic spaces and their complex and quaternionic counterparts. We deduce compactifications for Clifford–Klein forms of these homogeneous spaces, namely for quotients by discrete groups  $\Gamma$  acting properly discontinuously, in the case that  $\Gamma$  is word hyperbolic and acts via an Anosov representation. In particular, these Clifford–Klein forms are topologically tame.

## 1. Introduction

The goal of this note is twofold. First, we describe compactifications of certain families of reductive homogeneous spaces  $G/H$  by embedding  $G$  into a larger group  $G'$  and realizing  $G/H$  as a  $G$ -orbit in a flag manifold of  $G'$ . These homogeneous spaces include:

- group manifolds associated with classical Lie groups (Theorems 1.1 and 2.6; see also [He02]),
- certain affine symmetric spaces or reductive homogeneous spaces  $G/H$  given in Tables 2 and 3 (Propositions 1.5(1) and 5.8(1)),
- pseudo-Riemannian analogues of real hyperbolic spaces and their complex and quaternionic counterparts (see (1.3) in Section 1.4).

Second, we use these compactifications and a construction of domains of discontinuity from [GW12] to compactify Clifford–Klein forms of  $G/H$ , that is, quotient manifolds  $\Gamma \backslash G/H$ , in the case that  $\Gamma$  is a word hyperbolic group whose action on  $G/H$  is given by an Anosov representation  $\rho : \Gamma \rightarrow G \hookrightarrow G'$ . We deduce that these Clifford–Klein forms are topologically tame.

---

Received August 7, 2015. Revision received June 20, 2016.

FG and FK were partially supported by the Agence Nationale de la Recherche under the grant DiscGroup (ANR-11-BS01-013) and through the Labex CEMPI (ANR-11-LABX-0007-01). AW was partially supported by the National Science Foundation under agreement DMS-1536017, by the Sloan Foundation, by the Deutsche Forschungsgemeinschaft, by the European Research Council under ERC-Consolidator grant 614733, and by the Klaus Tschira Foundation. Part of this work was carried out while OG, FK, and AW were in residence at the MSRI in Berkeley, California, supported by the National Science Foundation under grant 0932078 000. The authors also acknowledge support from U.S. National Science Foundation grants DMS 1107452, 1107263, 1107367 “RNMS: GEometric structures And Representation varieties” (the GEAR Network).