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Potential Theory for Quaternionic Plurisubharmonic Functions

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Abstract. In this paper, we establish quaternionic versions of the potential description of various "small" sets related to quaternionic plurisubharmonic functions in the n-dimensional quaternionic space $\mathbb{H}^n$. We use the quaternionic capacity introduced in [31] to characterize the $(-\infty)$-sets of plurisubharmonic functions as the sets of vanishing capacity. The latter requirement is also equivalent to the negligibility of a set. We also prove the Josefson theorem on the equivalence of the locally and globally quaternionic polar sets in $\mathbb{H}^n$, following the Bedford–Taylor method.

1. Introduction

The pluripotential theory, which is a nonlinear complex counterpart of classical potential theory, has occupied an important place in mathematics. Although relatively young, the pluripotential theory has attracted considerable interest among analysts. The central part of the pluripotential theory is occupied by maximal plurisubharmonic functions and the generalized complex Monge–Ampère operator. Decisive progress in this field has been made by Bedford and Taylor [6; 7; 8; 9], Demailly [14; 15; 16; 17], Cegrell [10; 11; 12; 13], to mention a few. Cegrell's book [10] provides an excellent in-depth study of capacities in $\mathbb{C}^n$. See also [20] for a detailed discussion on various types of small sets in $\mathbb{C}^n$.

The potential theory for the Hessian equation has also been intensively studied in recent years. Labutin [21] studied the potential estimates for the real k-Hessian equation and used a special capacity to investigate the typical questions of potential theory: local behavior, removability of singularities, and polar, negligible, and thin sets. See [23; 24; 25; 26; 28] and references therein for other potential results for the complex Hessian and real Hessian equation.

In the n-dimensional quaternionic space $\mathbb{H}^n$, at present, little is known about the quaternionic pluripolar sets and the zero sets of the quaternionic capacities. The purpose of this paper is to give a potential-theoretic description of various "small" sets related to the quaternionic Monge–Ampère operator.

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