

# On Coxeter Mapping Classes and Fibered Alternating Links

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ABSTRACT. Alternating-sign Hopf plumbing along a tree yields fibered alternating links whose homological monodromy is, up to a sign, conjugate to some alternating-sign Coxeter transformation. Exploiting this tie, we obtain results about the location of zeros of the Alexander polynomial of the fibered link complement implying a strong case of Hoste’s conjecture, the trapezoidal conjecture, bi-orderability of the link group, and a sharp lower bound for the homological dilatation of the monodromy of the fibration. The results extend to more general hyperbolic fibered 3-manifolds associated with alternating-sign Coxeter graphs.

## 1. Introduction

In this paper, we study mapping classes defined by bipartite Coxeter graphs with sign-labels on the vertices determined by the bipartite structure. If the graph is connected and has at least two vertices, then these *alternating-sign Coxeter mapping classes* are pseudo-Anosov, and if the Coxeter graph is a tree, then the associated mapping class is the monodromy of an alternating fibered knot or link, which we call an (*alternating*) *Coxeter link*.

There has long been interest in the location of roots of Alexander polynomials for alternating links. Murasugi [18] showed that the coefficients of the polynomials have alternating signs, and hence no real root can be negative. Hoste conjectured that the real part of all zeros must be bounded from below by  $-1$ . This and related conjectures were settled for some classes of alternating links in [15; 13; 27; 7].

Using properties of alternating-sign Coxeter transformations, we give a simple proof that the roots of the Alexander polynomials for alternating Coxeter links are real and positive. By a result of Perron and Rolfsen [20] this implies that the fundamental group of the complement of an alternating Coxeter link is bi-orderable. Applying an interlacing property for alternating-sign Coxeter graphs, we show that the homological dilatations are monotone under graph inclusion. Thus, the minimum homological dilatation achieved by an alternating Coxeter link is  $\frac{3+\sqrt{5}}{2}$ , the square of the golden ratio. Similar properties hold for the Alexander polynomial of the mapping torus of alternating-sign Coxeter mapping classes.

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