Michigan Math. J. **65 (2016), 749–759**

The Cotangent Bundle of a Cominuscule Grassmannian

V. Lakshmibai, Vijay Ravikumar, & William Slofstra

Abstract. A theorem of the first author states that the cotangent bundle of the type A Grassmannian variety can be embedded as an open subset of a smooth Schubert variety in a two-step affine partial flag variety. We extend this result to cotangent bundles of cominuscule generalized Grassmannians of arbitrary Lie type.

1. Introduction

An earlier work of Lusztig and Strickland suggests possible connections between the conormal varieties to partial flag varieties on the one hand and affine Schubert varieties on the other. In particular, Lusztig [8] relates certain orbit closures arising from the type A cyclic quiver $\hat{A}_h$ to affine Schubert varieties. In the case $h = 2$, Strickland [11] relates such orbit closures to conormal varieties of determinantal varieties; furthermore, any determinantal variety can be canonically realized as an open subset of a Schubert variety in the Grassmannian [6].

Inspired by these results, the first author was interested in finding a relationship between affine Schubert varieties and conormal varieties to the Grassmannian. As a first step, she showed that the compactification of the cotangent bundle to the Grassmannian is canonically isomorphic to a Schubert variety in a two-step affine partial flag variety [5]. In this paper, we extend her result to cominuscule generalized Grassmannians of arbitrary finite type (such Grassmannians occur in types $A - E$).

1.1. Preliminaries

Let G_0 be a simple algebraic group over $\mathbb{C}$ with associated Lie algebra $\mathfrak{g}_0$ and simple roots $\{\alpha_1, \dots, \alpha_n\}$. A simple root α_i is *cominuscule* if the coefficient of α_i in any positive root of $\mathfrak{g}_0$ (written in the simple root basis) is less than or equal to 1.

The Weyl group of G_0 is generated by simple reflections $S_0 := \{s_1, \dots, s_n\}$ corresponding to the simple roots $\{\alpha_1, \dots, \alpha_n\}$. For any subset $K \subset S_0$, we let $P_K \subset G_0$ denote the parabolic subgroup whose Weyl group is generated by the elements of K. For $1 \le i \le n$, set $S_{0,i} := S_0 \setminus \{s_i\}$, so that $P_{S_{0,i}}$ is a maximal parabolic subgroup of G_0. The manifold $G_0/P_{S_{0,i}}$ is called a *generalized Grassmannian of type* G_0 and is said to be *cominuscule* if α_i is cominuscule. For additional background on cominuscule Grassmannians, see [1] or [7].

Received May 30, 2015. Revision received August 21, 2016.