Michigan Math. J. **65 (2016), 573–597**

A Construction of Slice Knots via Annulus Twists

Tetsuya Abe & Motoo Tange

Abstract. We give a new construction of slice knots via annulus twists. The simplest slice knots obtained by our method are those constructed by Omae. In this paper, we introduce a sufficient condition for given slice knots to be ribbon and prove that all Omae's knots are ribbon.

1. Introduction

The annulus twist is a certain operation on knots along an annulus embedded in the 3-sphere S^3. Osoinach [Os] found that this operation is useful in the study of 3-manifolds. Using annulus twists, he gave the first example of a 3-manifold admitting infinitely many presentations by 0-framed knots. For more studies, see [AJOT; AJLO; BGL; K; Tak; Te; Om].

Recently, the first author, Jong, Omae, and Takeuchi [AJOT] constructed knots related to the slice-ribbon conjecture: Let $K \subset S^3$ be a slice knot admitting an annulus presentation (for the definition, see Section 2), and K_n ($n \in \mathbb{Z}$) the knot obtained from K by the n-fold annulus twist. They proved that K_n bounds a smoothly embedded disk in a certain homotopy 4-ball $W(K_n)$ with $\partial W(K_n) \approx S^3$. A natural question is the following:

Question. Is $W(K_n)$ diffeomorphic to the standard 4-ball B^4?

If $W(K_n)$ is not diffeomorphic to B^4, then the homotopy 4-sphere obtained by capping it off is a counterexample of the smooth four-dimensional Poincaré conjecture. For related studies, see [A1; A2; FGMW; G1; G2; N; NS; Tan]. Our first result is the following.

Theorem 3.1. *Let K be a slice knot admitting an annulus presentation, and K_n ($n \in \mathbb{Z}$) the knot obtained from K by the n-fold annulus twist. Then the homotopy 4-ball $W(K_n)$ associated to K_n is diffeomorphic to B^4, that is,*

$$W(K_n) \approx B^4.$$

In particular, K_n is a slice knot.

The slice knots constructed in Theorem 3.1 are relevant to the slice-ribbon conjecture. Recall that a knot K in $S^3 = \partial B^4$ is called *slice* if it bounds a smoothly embedded disk $D \subset B^4$, and the embedded disk $D \subset B^4$ is called a *slice disk* for K.

Received April 8, 2015. Revision received March 8, 2016.

The first author was supported by JSPS KAKENHI Grant Numbers 23840021 and 13J05998. The second author was supported by JSPS KAKENHI Grant Number 24840006.