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Moriwaki Divisors and the Augmented Base Loci of Divisors on the Moduli Space of Curves

Salvatore Cacciola, Angelo Felice Lopez, & Filippo Viviani

Abstract. We study the cone of Moriwaki divisors on $\overline{M}_g$ by means of augmented base loci. Using a result of Moriwaki, we prove that an $\mathbb{R}$-divisor D satisfies the strict Moriwaki inequalities if and only if $\mathbf{B}_+(D) \subseteq \partial\overline{M}_g$. Then we draw some interesting consequences on the Zariski decomposition of divisors on $\overline{M}_g$, on the minimal model program of $\overline{M}_g$, and on the log canonical models $\overline{M}_g(\alpha)$.

1. Introduction

Let $g \geq 3$, and let $\overline{M}_g$ be the moduli space of stable curves of genus g. A striking result of Gibney, Keel and Morrison [GKM, Thm. 0.9] asserts that any nef divisor on $\overline{M}_g$, not linearly equivalent to zero, must be big. In terms of cones of divisors in the Néron–Severi space $N^1(\overline{M}_g)_{\mathbb{R}}$, this implies that the nef cone does not meet the boundary of the big cone along rational nonzero classes. As a matter of fact, as we shall see, the same is true for real classes: $\mathrm{Nef}(\overline{M}_g) - \{0\} \subset \mathrm{Big}(\overline{M}_g)$. One way to see this is to consider the *Moriwaki cone* $\mathrm{Mor}(\overline{M}_g)$, that is, the cone of $\mathbb{R}$-divisors D on $\overline{M}_g$ that are nef away from the boundary. The cone $\mathrm{Mor}(\overline{M}_g)$ was explicitly described by Moriwaki [M, Cor. 4.3] in terms of the generators $\lambda, \delta_0, \ldots, \delta_{\lfloor g/2 \rfloor}$: an $\mathbb{R}$-divisor $D \sim a\lambda - b_0\delta_0 - \cdots - b_{\lfloor g/2 \rfloor}\delta_{\lfloor g/2 \rfloor}$ belongs to $\mathrm{Mor}(\overline{M}_g)$ if and only if it is an *M-divisor,* that is, it satisfies the *Moriwaki inequalities*

$$a \geq 0, \qquad a \geq \frac{8g+4}{g} b_0, \qquad a \geq \frac{2g+1}{i(g-i)} b_i \quad \text{for all } i = 1, \ldots, \lfloor g/2 \rfloor. \tag{1}$$

The starting idea of this paper is that both the Moriwaki cone and its interior, that is, the cone of those $\mathbb{R}$-divisors that satisfy the strict Moriwaki inequalities and that we call *strict M-divisors,* can be interpreted in terms of restricted and augmented base loci.

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