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A Characterization of Singular-Hyperbolicity

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1. Introduction

The relationship between dominated splittings and uniform hyperbolicity was explored by Mañé in his solution of the stability conjecture for diffeomorphisms [18]. Pujals and Sambarino [22] studied it in their nowadays famous Theorem B: For C^2 surface diffeomorphisms, every compact invariant set with a dominated splitting whose periodic points are all hyperbolic saddle splits into a hyperbolic set and finitely many disjoint normally hyperbolic irrational circles. A similar relationship but between dominated splitting *with respect to the linear Poincaré flow* and uniform hyperbolicity was obtained by Aubin and Hertz [6]. Indeed, they proved that every nonsingular compact invariant set exhibiting a dominated splitting with respect to the Poincaré flow and whose periodic points are all hyperbolic saddle splits in a hyperbolic set and finitely many disjoint normally hyperbolic irrational tori. In light of these results, it is natural to think about the singular case, namely, is it possible to obtain a similar decomposition for compact invariant sets with singularities whose nonsingular points exhibit a dominated splitting with respect to the linear Poincaré flow and whose periodic points are all hyperbolic of saddle type? However, this kind of question must face the problem of a natural candidate for uniform hyperbolicity. Indeed, the *geometric Lorenz attractor* [14] is a nonhyperbolic compact invariant set of a C^∞ three-dimensional flow for which the periodic points are all hyperbolic saddle, has no irrational tori, and, nevertheless, its nonsingular points exhibit a dominated splitting with respect to the linear Poincaré flow. The notion of *singular-hyperbolicity* emerges as this candidate, the geometric Lorenz attractor as well as any robustly transitive attractor with singularities of a three-dimensional flow enjoy it [20]. It is then natural to ask if there is a relationship between dominated splittings with respect to the linear Poincaré flow and singular-hyperbolicity, namely, if for every C^2 three-dimensional flow, every compact invariant set whose nonsingular points exhibit a dominated splitting *with respect to the linear Poincaré flow* and whose periodic points are all hyperbolic saddle splits into a singular-hyperbolic set for the flow, a singular-hyperbolic set for the reversed flow, and finitely many disjoint normally hyperbolic irrational tori. In this scenario, Crovisier and Yang announced recently

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