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Multiplier Ideals in Two-Dimensional Local Rings with Rational Singularities

Maria Alberich-Carramiñana,
Josep Àlvarez Montaner, & Ferran Dachs-Cadefau

Abstract. The aim of this paper is to study jumping numbers and multiplier ideals of any ideal in a two-dimensional local ring with a rational singularity. In particular, we reveal which information encoded in a multiplier ideal determines the next jumping number. This leads to an algorithm to compute sequentially the jumping numbers and the whole chain of multiplier ideals in any desired range. As a consequence of our method, we develop the notion of *jumping divisor* that allows us to describe the jump between two consecutive multiplier ideals. In particular, we find a unique minimal jumping divisor that is studied extensively.

1. Introduction

Let X be a complex algebraic variety with mild singularities, and $\mathcal{O}_{X,O}$ the local ring of a point $O \in X$. To any ideal $\mathfrak{a} \subseteq \mathcal{O}_{X,O}$ we may associate a family of *multiplier ideals* $\mathcal{J}(\mathfrak{a}^{\lambda})$ parameterized by positive rational numbers $\lambda \in \mathbb{Q}_{>0}$. Indeed, they form a nested sequence of ideals

$$\mathcal{O}_{X,O} \supsetneq \mathcal{J}(\mathfrak{a}^{\lambda_1}) \supsetneq \mathcal{J}(\mathfrak{a}^{\lambda_2}) \supsetneq \cdots \supsetneq \mathcal{J}(\mathfrak{a}^{\lambda_i}) \supsetneq \cdots,$$

and the rational numbers $0 < \lambda_1 < \lambda_2 < \cdots$ where the multiplier ideals change are called *jumping numbers*. The first jumping number λ_1 is also known as the *log-canonical threshold*. Multiplier ideals and their associated jumping numbers have proven to be a powerful tool to understand the geometry of singularities. They are defined using a log-resolution of the pair $(X, \mathfrak{a})$. In fact, smaller or more dense jumping numbers can be thought to correspond to "worse" singularities.

The aim of this paper is to present a new approach to the understanding of multiplier ideals and jumping numbers of any ideal $\mathfrak{a}$ in the local ring $\mathcal{O}_{X,O}$ of a complex surface X having at worst a rational singularity at O. This is a case, especially where X is smooth, that has received a lot of attention in recent years because of the interesting properties these invariants satisfy (see the works of

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