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# Seidel Elements and Mirror Transformations for Toric Stacks

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Abstract. We give a precise relation between the mirror transformation and the Seidel elements for weak Fano toric Deligne–Mumford stacks. Our result generalizes the corresponding result for toric varieties proved by González and Iritani [5]. The correction coefficients that we computed match with the instanton corrections from genus 0 open Gromov–Witten invariants for toric Calabi–Yau orbifolds in [3].

## 1. Introduction

In [5], González and Iritani gave a precise relation between the mirror map and the Seidel elements for a smooth projective weak Fano toric variety $X$. The goal of this paper is to generalize the main theorem of [5] to a smooth projective weak Fano toric Deligne–Mumford stack $\mathcal{X}$.

Let $\mathcal{X}$ be a smooth projective weak Fano toric Deligne–Mumford stack. The mirror theorem can be stated as an equality between the $I$-function and the $J$-function via a change of coordinates, called mirror map (or mirror transformation). We refer to [4] and Section 4.1 of [6] for further discussions.

Let $Y$ be a monotone symplectic manifold. For a loop $\lambda$ in the group of Hamiltonian symplectomorphisms on $Y$, Seidel [10] constructed an invertible element $S(\lambda)$ in (small) quantum cohomology counting sections of the associated Hamiltonian $Y$-bundle $E_\lambda \to \mathbb{P}^1$. The Seidel element $S(\lambda)$ defines an element in $\mathrm{Aut}(QH(Y))$ via quantum multiplication, and the map $\lambda \mapsto S(\lambda)$ gives a representation of $\pi_1(\mathrm{Ham}(Y))$ on $QH(Y)$. McDuff and Tolman [9] extended this construction to all symplectic manifolds. The definition of Seidel representation and Seidel element were extended to symplectic orbifolds by Tseng and Wang [11].

Let $D_1, \dots, D_m$ be the classes in $H^2(X)$ Poincaré dual to the toric divisors. When the loop $\lambda$ is a circle action, McDuff and Tolman [9] considered the Seidel element $\tilde{S}_j$ associated to an action $\lambda_j$ that fixes the toric divisor $D_j$. Given a circle action on $X$ (resp. $\mathcal{X}$), the Seidel element in [5] (resp. [11]) is defined using the small quantum cohomology ring. In this paper, we need to define it, for smooth projective Deligne–Mumford stack, with deformed quantum cohomology to include the bulk deformations. For weak Fano toric Deligne–Mumford stack, the mirror theorem in [6] shows that the mirror map $\tau(y) \in H^{\le 2}_{\mathrm{orb}}(\mathcal{X})$; therefore, we will only need bulk deformations with $\tau \in H^{\le 2}_{\mathrm{orb}}(\mathcal{X})$.

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