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# The Representations of the Automorphism Groups and the Frobenius Invariants of K3 Surfaces

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Abstract. For a complex algebraic K3 surface, it is known that the representations of the automorphism group on the transcendental cycles is finite and is isomorphic to the representation on the two-forms. In this paper, we prove similar results for a K3 surface defined over a field of odd characteristic. Also, we prove that the height and the Artin invariant of a K3 surface equipped with a nonsymplectic automorphism of some high order are determined by a congruence class of the base characteristic.

## 1. Introduction

When $X$ is an algebraic complex K3 surface, the second integral singular cohomology $H^2(X,\mathbb{Z})$ is a free Abelian group of rank 22 equipped with a lattice structure isomorphic to $U^3 \oplus E_8^2$. Here $U$ is the hyperbolic plane, and $E_8$ is the unique unimodular, even, and negative definite lattice of rank 8. The cycle map gives a primitive embedding of the Neron–Severi group of $X$ into the second cohomology $NS(X) \hookrightarrow H^2(X,\mathbb{Z})$. The rank of $NS(X)$ is called the Picard number of $X$ and is denoted by $\rho(X)$. The orthogonal complement of this embedding is called the transcendental lattice of $X$ and is denoted by $T(X)$. The rank of the transcendental lattice is $22-\rho(X)$. Cohomology $H^2(X,\mathbb{Z})$ is an overlattice of $NS(X) \oplus T(X)$, and

$$|H^2(X,\mathbb{Z})/(NS(X) \oplus T(X))| = |d(NS(X))|.$$

The one-dimensional complex space of global holomorphic two-forms of $X$, $H^0(X,\Omega^2_{X,\mathbb{C}})$ is a direct factor of $H^2(X,\mathbb{Z}) \otimes \mathbb{C} = H^2(X,\mathbb{C})$, and by the Lefschetz (1, 1) theorem,

$$NS(X) = H^0(X,\Omega^2_{X,\mathbb{C}})^{\perp} \cap H^2(X,\mathbb{Z})$$

in $H^2(X,\mathbb{C})$. In particular, $H^0(X,\Omega^2_{X/\mathbb{C}})$ is a direct factor of $T(X) \otimes \mathbb{C}$. The automorphism group of $X$, $\mathrm{Aut}(X)$, has natural actions on $T(X)$ and on $H^0(X,\Omega^2_{X/\mathbb{C}})$. Let us denote the actions of $\mathrm{Aut}(X)$ on the transcendental lattice and the two-forms by

$$\chi_X : \mathrm{Aut}(X) \to O(T(X)) \quad \text{and} \quad \rho_X : \mathrm{Aut}(X) \to Gl(H^0(X,\Omega^2_{X/\mathbb{C}})).$$

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