Michigan Math. J. 65 (2016), 57–87

# $F$-pure Thresholds of Homogeneous Polynomials

Daniel J. Hernández, Luis Núñez-Betancourt, Emily E. Witt, & Wenliang Zhang

Abstract. We characterize $F$-pure thresholds of polynomials that are homogeneous under some $\mathbb{N}$-grading and have an isolated singularity at the origin. Our description places rigid restrictions on these invariants and allows us to produce finite lists of possible values of such $F$-pure thresholds; these lists are often minimal, and in specific examples, may even allow us to exactly determine the value of the $F$-pure threshold in question. The result, when combined with other techniques, sheds further light on the relationship between $F$-pure and log canonical thresholds in our setting. We compute uniform bounds for the difference between $F$-pure and log canonical thresholds established by Mustaţă and the fourth author and examine the set of primes for which the $F$-pure and log canonical threshold of a polynomial must differ. Moreover, we establish a specific subcase of the ACC conjecture for $F$-pure thresholds and provide further supporting evidence for this conjecture.

## 1. Introduction

The $F$-pure threshold, first defined in [TW04, Def. 2.1], is a numerical invariant of singularities in positive characteristic defined via the Frobenius (or $p$-th power) endomorphism; though they can be defined more generally, we will only consider $F$-pure thresholds of polynomials over fields of prime characteristic and thus follow the treatment given in [MTW05]. The $F$-pure threshold of such a polynomial $f$, denoted $\operatorname{fpt}(f)$, is always a rational number in $(0, 1]$, with smaller values corresponding to "worse" singularities [BMS08; BMS09; B+09].

The log canonical threshold of a polynomial $f_{\mathbb{Q}}$ over $\mathbb{Q}$, denoted $\operatorname{lct}(f_{\mathbb{Q}})$, is also a numerical invariant measuring the singularities of $f_{\mathbb{Q}}$ and can be defined via integrability conditions, or, more generally, via resolution of singularities; like the $F$-pure threshold, $\operatorname{lct}(f_{\mathbb{Q}})$ is a rational number in $(0, 1]$; see [BL04] for more on this and on related invariants. The connections between $F$-pure and log canonical thresholds run deep: Since any $\frac{a}{b} \in \mathbb{Q}$ determines a well-defined element of

Received July 2, 2014. Revision received September 14, 2015.

The first author gratefully acknowledges support from the Ford Foundation (FF) through an FF Postdoctoral Fellowship. The second author thanks the National Council of Science and Technology (CONACyT) of Mexico for support through Grant #210916. The fourth author was partially supported by NSF grants DMS #1247354 and DMS #1068946, and a Nebraska EPSCoR First Award. This collaboration began during visits supported by a travel grant from the AMS Mathematical Research Communities 2010 Commutative Algebra program.