Michigan Math. J. 65 (2016), 3–33

On IHS Fourfolds with $b_2 = 23$

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ABSTRACT. The present work is concerned with the study of four-dimensional irreducible holomorphic symplectic manifolds with second Betti number 23. We describe their birational geometry and their relations to EPW sextics.

1. Introduction

By an *irreducible holomorphic symplectic* (*IHS*) *fourfold* we mean (see [B1]) a four-dimensional simply connected Kähler manifold with trivial canonical bundle that admits a unique (up to a constant) closed nondegenerate holomorphic 2-form and is not a product of two manifolds. These manifolds are among the building blocks of Kähler fourfolds with trivial first Chern class [B1, Thm. 2]. In the case of four-dimensional examples their second Betti number b_2 is bounded, and $3 \leq b_2 \leq 8$ or $b_2 = 23$ (see [Gu]). There are however only two known families of IHSs in this dimension, one with $b_2 = 7$ and the other with $b_2 = 23$ [B1]. The first is the deformation of the Hilbert scheme of two points on a K3 surface, and the second is the deformation of the Hilbert scheme of three points that sum to 0 on an Abelian surface.

In this paper we address the problem of classification of IHS fourfolds X with $b_2 = 23$. This program was initiated by O'Grady, whose purpose is to prove that IHS fourfolds that are numerically equivalent to the Hilbert scheme of two points on a K3 surface are deformation equivalent to this Hilbert scheme (are of Type $K3^{[2]}$).

It is known from [V] and [Gu] that for IHS fourfolds with $b_2 = 23$, the cup product induces an isomorphism

$$\mathrm{Sym}^2 H^2(X, \mathbb{Q}) \simeq H^4(X, \mathbb{Q}) \tag{1.1}$$

Received April 21, 2014. Revision received October 6, 2015.

The project was supported by MNSiW, N N201 414539. The Appendix is supported by the program Iuventus plus “Rangi i rangi brzegowe wielomianów oraz równania rozmaitości siecznych”.