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Value Distribution of Holomorphic Curves on an Angular Domain

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Abstract. In this paper, we investigate the value distribution of holomorphic curves on an angular domain from the point of view of potential theory and establish the first and second fundamental theorems corresponding to those theorems of Ahlfors–Shimizu, Nevanlinna, and Tsuji on meromorphic functions in an angular domain, which have not been established before in other references. As applications of these theorems, we introduce the singular directions of holomorphic curves and prove their existences and investigate the growth of holomorphic curves with radially distributed hyperplanes and uniqueness of holomorphic curves in an angular domain. The obtained results are transferred to the algebroid functions.

1. Introduction

For $0 \le \alpha < \beta \le 2\pi$, by $\Omega(\alpha, \beta)$ we denote the angular domain

$$\Omega(\alpha, \beta) := \{z : \alpha < \arg z < \beta\}$$

and by $\bar{\Omega}(\alpha, \beta)$ its closure. Sometimes, without occurrence of any confusion in the context, we write simply Ω instead of $\Omega(\alpha, \beta)$. The behavior of a function meromorphic in an angle has been investigated in many references, such as [23; 2; 15; 4], and [31]. For the purpose of studying the topic, characteristic functions that describe the growth of the meromorphic functions in an angle have been introduced; for instance, the Ahlfors–Shimizu characteristic, Nevanlinna characteristic, and Tsuji characteristic for an angle, and the corresponding main theorems have been established. The second main theorem for the Ahlfors–Shimizu characteristic for an angle can be found in [29], and in [12] for the Nevanlinna characteristic and Tsuji characteristic for an angle. These main theorems were collected and compared to each other in [31].

We emphasize the usage of the Ahlfors–Shimizu second main theorem for an angle in proving the existence of T-direction of a meromorphic function on the complex plane; see [31] for a detailed discussion on singular directions. We remark that $\Omega_\varepsilon = \Omega(\alpha + \varepsilon, \beta - \varepsilon)$ in the theorem cannot be replaced by Ω.

The Nevanlinna second main theorem for an angle was used in [12; 10; 26; 30], and [31] to investigate the growth of meromorphic functions with some radially distributed values. The usage of the second main theorem produces a basic and

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