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Brauer Groups of Quot Schemes

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Abstract. Let X be an irreducible smooth complex projective curve. Let $\mathcal{Q}(r,d)$ be the Quot scheme parameterizing all coherent subsheaves of $\mathcal{O}_X^{\oplus r}$ of rank r and degree $-d$. There are natural morphisms $\mathcal{Q}(r,d) \longrightarrow \mathrm{Sym}^d(X)$ and $\mathrm{Sym}^d(X) \longrightarrow \mathrm{Pic}^d(X)$. We prove that both these morphisms induce isomorphism of Brauer groups if $d \geq 2$. Consequently, the Brauer group of $\mathcal{Q}(r,d)$ is identified with the Brauer group of $\mathrm{Pic}^d(X)$ if $d \geq 2$.

1. Introduction

Let X be an irreducible smooth projective curve defined over $\mathbb{C}$. For any integer $r \geq 1$, consider the trivial holomorphic vector bundle $\mathcal{O}_X^{\oplus r}$ on X. For any $d \geq 0$, let $\mathcal{Q}(r,d)$ denote the Quot scheme that parameterizes all torsion quotients of degree d of the $\mathcal{O}_X$-module $\mathcal{O}_X^{\oplus r}$. This $\mathcal{Q}(r,d)$ is an irreducible smooth complex projective variety of dimension rd.

For every $Q \in \mathcal{Q}(r,d)$, we have a corresponding short exact sequence

$$0 \longrightarrow \mathcal{F}(Q) \stackrel{\rho}{\longrightarrow} \mathcal{O}_X^{\oplus r} \longrightarrow Q \longrightarrow 0.$$

The pairs $(\mathcal{O}_X^{\oplus r})^* = \mathcal{O}_X^{\oplus r} \stackrel{\rho^*}{\longrightarrow} \mathcal{F}(Q)^*$ are vortices of a particular numerical type. The Quot scheme $\mathcal{Q}(r,d)$ is a moduli space of vortices of a particular numerical type (see [BDW; Ba; BR], and references therein).

Sending such Q to the scheme theoretic support of the quotient for the homomorphism

$$\bigwedge^r \mathcal{F}(Q) \longrightarrow \bigwedge^r \mathcal{O}_X^{\oplus r}$$

induced by the inclusion $\mathcal{F}(Q) \longrightarrow \mathcal{O}_X^{\oplus r}$, we get a morphism

$$\varphi : \mathcal{Q}(r,d) \longrightarrow \mathrm{Sym}^d(X).$$

Sending any $Q \in \mathcal{Q}(r,d)$ to the holomorphic line bundle $\bigwedge^r \mathcal{F}(Q)^*$, we get a morphism

$$\varphi' : \mathcal{Q}(r,d) \longrightarrow \mathcal{Q}(1,d) = \mathrm{Pic}^d(X).$$

On the other hand, we have the morphism

$$\xi_d : \mathrm{Sym}^d(X) \longrightarrow \mathrm{Pic}^d(X)$$

that sends any $(x_1,\dots,x_d)$ to the holomorphic line bundle $\mathcal{O}_X(\sum_{i=1}^d x_i)$. Note that $\varphi' = \xi_d \circ \varphi$.

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