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An Example Concerning Holomorphicity of Meromorphic Mappings Along Real Hypersurfaces

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Abstract. We construct an example of a rational mapping F from $\mathbb{C}^2$ to $\mathbb{P}^2$ that has indeterminacies on the unit sphere $\mathbb{S}^3 \subset \mathbb{C}^2$ such that $F|_{\mathbb{S}^3}$ is continuous, the image $K := F(\mathbb{S}^3)$ is contained in the affine part $\mathbb{C}^2$ of $\mathbb{P}^2$, and K does not contain any germ of a nonconstant complex curve.

1. Introduction

Let $F : U \to \mathbb{P}^N$ be a meromorphic mapping from a domain $U \subset \mathbb{C}^n$ to the complex projective space. Here we always suppose that $n \geq 2$ and $N \geq 2$. Denote by I_F the set of points of indeterminacy of F, that is, $z_0 \in I_F$ if and only if F is not holomorphic in any neighborhood of z_0. As it is well known, I_F is an analytic subset of U of codimension at least two. Recall that the *full image* by F of a point z_0, denoted as $F[z_0]$, is the set of all cluster points of F at z_0, that is,

$$F[z_0] = \{x \in \mathbb{P}^N : \exists z_k \to z_0, z_k \notin I_F \text{ such that } F(z_k) \to x\}. \tag{1.1}$$

The mapping F is holomorphic in a neighborhood of z_0 if and only if $F[z_0]$ is a singleton. Likewise, we can define the full image of z_0 by F along a closed subset M accumulating to z_0; for example, M can be a complex curve or a real hypersurface containing z_0:

$$F_M[z_0] = \{x \in \mathbb{P}^N : \exists m_k \in M \setminus I_F, m_k \to z_0 \text{ such that } F(m_k) \to x\}. \tag{1.2}$$

If M is a complex curve, then $F_M[z_0]$ is always a singleton, regardless of whether z_0 is a regular or an indeterminacy point of F. As we shall see in our example, the same can happen if M occurs to be a real hypersurface with z_0 being an indeterminacy point of F. The proper image or transform of a closed subset $M \subset U$ under F is defined now to be the union of full images along M of its points. In other words,

$$F_M[M] = \bigcup_{m \in M} F_M[m]. \tag{1.3}$$

If M is compact, for example, $M = \mathbb{S}^3 \subset \mathbb{C}^2 = U$, then

$$F_M[M] = \overline{F(M \setminus I_F)}. \tag{1.4}$$

We denote as w_1, w_2 the standard coordinates in $\mathbb{C}^2$; $\mathbb{S}^3 = \{(w_1, w_2) : |w_1|^2 + |w_2|^2 = 1\}$ stands for the unit sphere in $\mathbb{C}^2$, and $\mathbb{B}^2$ for the *open* unit ball. Our goal in this note is to construct the following example.

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