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General Hilbert Stacks and Quot Schemes

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In memory of Dan Laksov

ABSTRACT. We prove the algebraicity of the Hilbert functor, the Hilbert stack, the Quot functor, and the stack of coherent sheaves on an algebraic stack X with (quasi-)finite diagonal without any finiteness assumptions on X. We also give similar results for Hom stacks and Weil restrictions.

Introduction

Let S be a scheme, and let $f : X \to S$ be a morphism between algebraic stacks that is locally of finite presentation. If f is separated, then it is well known that the Hilbert functor $\mathcal{H}ilb_{X/S}$ is an algebraic space, locally of finite presentation over S [Art69; OS03; Ols05]. If f is not separated but has a quasi-compact and separated diagonal with affine stabilizers, then one can instead prove that the Hilbert stack $\mathscr{H}^{\mathrm{qfin}}_{X/S}$—parameterizing proper flat families with a quasi-finite morphism to X—is an algebraic stack, locally of finite presentation over S [HR14; Ryd11]. The first main result of this paper is a partial generalization of these two results to stacks that are not locally of finite presentation.

THEOREM A. *Let S be a scheme, and let X be an algebraic stack over S.*

(i) *If $X \to S$ has a finite diagonal, then $\mathcal{H}ilb_{X/S}$ is a separated algebraic space, and $\mathscr{H}^{\mathrm{qfin}}_{X/S}$ is an algebraic stack with affine diagonal.*
(ii) *If $X \to S$ has a quasi-compact and separated diagonal with affine stabilizers, then $\mathscr{H}^{\mathrm{qfin}}_{X/S}$ is an algebraic stack with quasi-affine diagonal.*

In particular, if X is any separated scheme, algebraic space, or Deligne–Mumford stack, then $\mathcal{H}ilb_{X/S}$ is an algebraic space.

Our second result is about stacks of sheaves. Let us again first recall the classical situation. So, let $f : X \to S$ be a separated morphism between algebraic stacks that is locally of finite presentation. Then $\mathscr{C}oh(X/S)$—the stack of finitely presented sheaves on X that are flat and proper over S—is an algebraic stack, locally of finite presentation over S with affine diagonal [Lie06, Thm. 2.1], [Hal14b, Thm. 8.1]. If we are also given a quasi-coherent sheaf $\mathcal{F}$ on X, then $\mathrm{Quot}(X/S, \mathcal{F})$ is a separated algebraic space [Hal14b, Cor. 8.2]. Usually, it is

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