

A Generalization of Sankaran and LVMB Manifolds

LAURENT BATTISTI & KARL OELJEKLAUS

ABSTRACT. We describe the construction of a new class of non-Kähler compact complex manifolds. They can be seen as a generalization of Sankaran, OT, and LVMB manifolds. Moreover, we give properties of these new spaces. Their Kodaira dimension is $-\infty$, and under a mild condition, their algebraic dimension is equal to zero.

0. Introduction

In this article, we construct a new family of non-Kähler complex compact manifolds by a combination of methods of Bosio [5] and Sankaran [12]. The class of manifolds constructed in this paper appears as a generalization of already known examples of non-Kähler manifolds, namely LVMB and Sankaran manifolds (ibid) along with OT manifolds [9].

Although the field of non-Kähler geometry remains relatively unexplored, there is, nevertheless, a constant progress. New classes of non-Kähler compact complex manifolds have been constructed and studied recently.

The first example is given by the class of LVMB manifolds. Their construction is due to Bosio [5] and can be summarized as follows. Given a family of subsets of $\{0, \dots, n\}$ all having $2m + 1$ elements (where n and m are integers such that $2m \leq n$) and a family of $n + 1$ linear forms on \mathbb{C}^m satisfying technical conditions, we can find an open subset U of $\mathbb{P}_n(\mathbb{C})$ and an action of a complex Lie group $G \cong \mathbb{C}^m$ on $\mathbb{P}_n(\mathbb{C})$ such that the quotient U/G is a compact complex manifold. These manifolds generalize Hopf and Calabi–Eckmann manifolds, and they also generalize a class of manifolds due to Meersseman [8], called LVM manifolds.

OT manifolds were constructed in [9] by the second author and M. Toma. We start by choosing an algebraic number field K having $s > 0$ (resp. $2t > 0$) real (resp. complex) embeddings. Then, for a nice choice of a subgroup A of the groups of units \mathcal{O}_K^* of K , the quotient $X(K, A)$ of $\mathbb{H}^s \times \mathbb{C}^t$ under the action of $A \times \mathcal{O}_K$ is a complex compact manifold. The required condition on A is that the projection on the first s coordinates of its image through the logarithmic representation of units

$$\begin{aligned} \ell : \mathcal{O}_K^* &\longrightarrow \mathbb{R}^{s+t}, \\ a &\longmapsto (\ln |\sigma_1(a)|, \dots, \ln |\sigma_s(a)|, 2 \ln |\sigma_{s+1}(a)|, \dots, 2 \ln |\sigma_{s+t}(a)|) \end{aligned} \tag{1}$$

Received May 5, 2014. Revision received September 18, 2014.

The second author is partially supported by the ANR project MNGNK, decision #ANR-10-BLAN-0118.