Michigan Math. J. 64 (2015), 77–108

On the Geometry of Abel Maps for Nodal Curves

Alex Abreu, Juliana Coelho, & Marco Pacini

Abstract. In this paper, we give local conditions to the existence of Abel maps for smoothings of nodal curves extending the Abel maps for the generic fiber. We use this result to construct Abel maps of any degree for nodal curves with two components.

1. Introduction

1.1. History

Let C be a smooth projective curve over an algebraically closed field and fix a point P in C. A degree-d Abel map is a map $\alpha_L^d : C^d \to J_C$ from the product of d copies of C to its Jacobian J_C, sending $(Q_1, \dots, Q_d)$ to the invertible sheaf $L(dP - Q_1 - \cdots - Q_d)$, where L is an invertible sheaf on C. It is classically known that this map encodes many geometric properties of the curve C. For instance, the Abel theorem states that the fibers of α_L^d are complete linear series on C, up to the action of the dth symmetric group. Thus, all possible embeddings of C in projective spaces are known once we know its Abel maps.

Often, to study linear series on smooth curves, we resort to degenerations to singular curves. Then, it is important to understand how linear series behave under such degenerations. It was through the study of these degenerations that Griffiths and Harris proved the celebrated Brill–Noether theorem in [14], and later Gieseker proved Petri's conjecture in [13]. This inspired the seminal work of Eisenbud and Harris [9], where they introduced the theory of limit linear series for curves of compact type. Nevertheless, a satisfactory general theory of limit linear series has not yet been obtained, although there are several works in this direction for curves with two components, for instance, Coppens and Gatto [8] and Esteves and Medeiros [11]. More recently, Osserman [15] gave a more refined notion of limit linear series for a curve of compact type with two components.

Since there is a relationship between linear series and Abel maps for smooth curves, an interplay between limit linear series and Abel maps for singular curves is expected. This interplay was explored by Esteves and Osserman [12] for curves of compact type with two components, for which natural Abel maps exist. However, Abel maps for singular curves have been constructed only in a few cases: for irreducible curves in [1], in degree one in [3] and [4], in degree two in [5], [6], [16], and [17], and for curves of compact type and in any degree, in [7].

Received December 10, 2013. Revision received August 1, 2014.
The third author was partially supported by CNPq, processo 300714/2010-6.