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Compact Bilinear Commutators: The Weighted Case

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Abstract. Commutators of bilinear Calderón–Zygmund operators and multiplication by functions in a certain subspace of the space of functions of bounded mean oscillation are shown to be compact on appropriate products of weighted Lebesgue spaces.

1. Introduction and Statements of Main Results

The study in harmonic analysis of commutators of singular integrals with pointwise multiplication by functions in *BMO* started with the by now well-known 1976 work of Coifman, Rochberg, and Weiss [6]. A couple of years later, in another classic work in the subject, Uchiyama [16] proved that the L^p-boundedness result in [6] can be refined to a compactness one if the space *BMO* is replaced by the smaller space *CMO*. Recently, Bényi and Torres [2] revisited a notion of compactness in a bilinear setting, which was first introduced by Calderón in his fundamental paper on interpolation [3]. They showed in [2] that commutators of bilinear Calderón–Zygmund operators with multiplication by *CMO* functions are compact bilinear operators from $L^{p_1} \times L^{p_2} \to L^p$ for $1 < p_1, p_2 < \infty$ and $1/p_1 + 1/p_2 = 1/p \leq 1$, thus giving an extension to the bilinear setting of result in [16] for the linear case. In a subsequent joint work with Damián and Moen [1], the scope of the notion of compactness was expanded to include the commutators of a larger family of operators that contains bilinear Calderón–Zygmund ones and several singular bilinear fractional integrals. All these compactness results rely on the Frechét–Kolmogorov–Riesz characterization of precompact sets in unweighted Lebesgues spaces L^p; see Yosida's book [17, p. 275] and the expository note of Hanche-Olsen and Holden [10].

What happens if we change the Lebesgue measure dx with weighted versions $w\,dx$? This article originates in this natural question. Although seemingly simple, the answer to this question turns out to be more delicate than in the unweighted case. As we shall see, the compactness on products of weighted Lebesgue spaces depends rather crucially on the class of weights w considered. We note that in the linear case the compactness of the commutator on weighted spaces was not

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