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# A Half-Space Theorem for Ideal Scherk Graphs in $M \times \mathbb{R}$

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Abstract. We prove a half-space theorem for an ideal Scherk graph $\Sigma \subset M \times \mathbb{R}$ over a polygonal domain $D \subset M$, where $M$ is a Hadamard surface whose curvature is bounded above by a negative constant. More precisely, we show that a properly immersed minimal surface contained in $D \times \mathbb{R}$ and disjoint from $\Sigma$ is a translate of $\Sigma$.

## 1. Introduction

A well-known result in the global theory for proper minimal surfaces in the Euclidean 3-space is the so-called *half-space theorem* due to Hoffman and Meeks [11], which says that if a properly immersed minimal surface $S$ in $\mathbb{R}^3$ lies on one side of some plane $P$, then $S$ is a plane parallel to $P$. They also proved the *strong half-space theorem:* two properly immersed minimal surfaces in $\mathbb{R}^3$ that do not intersect must be parallel planes.

The problem of giving conditions that force two minimal surfaces of a Riemannian manifold to intersect has received considerable attention, and many people have worked on this subject.

Notice that there is no half-space theorem in Euclidean spaces of dimensions greater than 4 since there exist rotational proper minimal hypersurfaces contained in a slab.

Similarly, there exists no half-space theorem for horizontal slices in $\mathbb{H}^2 \times \mathbb{R}$ since rotational minimal surfaces (catenoids) are contained in a slab [13; 14]. However, there are half-space theorems for constant mean curvature (CMC) $1/2$ surfaces in $\mathbb{H}^2 \times \mathbb{R}$ [10; 15]. For instance, Hauswirth, Rosenberg, and Spruck [10] proved that if $S$ is a properly immersed CMC $1/2$ surface in $\mathbb{H}^2 \times \mathbb{R}$, contained on the mean convex side of a horocylinder $C$, then $S$ is a horocylinder parallel to $C$; and if $S$ is embedded and contains a horocylinder $C$ on its mean convex side, then $S$ is also a horocylinder parallel to $C$. Nelli and Sa Earp [15] showed that in $\mathbb{H}^2 \times \mathbb{R}$ the mean convex side of a simply connected rotational CMC $1/2$ surface cannot contain a complete CMC $1/2$ surface besides the rotational simply connected ones.

Other examples of homogeneous manifolds where there are half-space theorems for minimal surfaces are $\mathrm{Nil}_3$ and $\mathrm{Sol}_3$ [1; 4; 5]. For instance, we know that if a properly immersed minimal surface $S$ in $\mathrm{Nil}_3$ lies on one side of some entire minimal graph $\Sigma$, then $S$ is the image of $\Sigma$ by a vertical translation.

Mazet [12] proved a general half-space theorem for constant mean curvature surfaces. Under certain hypothesis, he proved that in a Riemannian 3-manifold of

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