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Mirror Symmetry for Stable Quotients Invariants

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Abstract. The moduli space of stable quotients introduced by Marian, Oprea, and Pandharipande provides a natural compactification of the space of morphisms from nonsingular curves to a nonsingular projective variety and carries a natural virtual class. We show that the analogue of Givental's J-function for the resulting twisted projective invariants is described by the same mirror hypergeometric series as the corresponding Gromov–Witten invariants (which arise from the moduli space of stable maps), but without the mirror transform (in the Calabi–Yau case). This implies that the stable quotients and Gromov–Witten twisted invariants agree if there is enough "positivity," but not in all cases. As a corollary of the proof, we show that certain twisted Hurwitz numbers arising in the stable quotients theory are also described by a fundamental object associated with this hypergeometric series. We thus completely answer some of the questions posed by Marian, Oprea, and Pandharipande concerning their invariants. Our results suggest a deep connection between the stable quotients invariants of complete intersections and the geometry of the mirror families. As in Gromov–Witten theory, computing Givental's J-function (essentially a generating function for genus 0 invariants with one marked point) is key to computing stable quotients invariants of higher genus and with more marked points; we exploit this in forthcoming papers.

1. Introduction

Gromov–Witten invariants of a smooth projective variety X are certain counts of curves in X that arise from integrating against the virtual class of the moduli space of stable maps. These are known to possess striking structures, which are often completely unexpected from the classical point of view. For example, the genus 0 Gromov–Witten invariants of a quintic threefold, that is, a degree 5 hypersurface in $\mathbb{P}^4$, are related by a so-called mirror formula to a certain hypergeometric series. This relation was explicitly predicted in [2] and mathematically confirmed in [8] and [13] in the 1990s. In fact, the prediction of [2] has been shown to be a special case of mirror symmetry for certain twisted Gromov–Witten invariants of projective complete intersections of sufficiently small total multidegree [7; 14]; these invariants are associated with direct sums of line bundles (positive and negative) over $\mathbb{P}^n$. This relation is often described by assembling two-point Gromov–Witten invariants (but without constraints on the second marked point) into a generating

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