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Factorization of p-Dominated Polynomials through L^p-Spaces

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Introduction

Since Pietsch's seminal paper [32], the study of the nonlinear operators ideals has been strongly developed. Several polynomial ideals have been defined and studied, and it can be said that it is now a relevant topic in functional analysis and important contributions have been made by many mathematicians such as R. Alencar, R. M. Aron, G. Botelho, V. Dimant, M. Matos, Y. Meléndez, D. Pellegrino, J. Santos, J. Seoane-Sepúlveda, and A. Tonge, among others. The hope of these nonlinear ideals is that they keep the main properties of the corresponding linear operator ideals, and so the linear theory can be lifted and extended to more general settings. To establish the relationship between an operator ideal and its natural polynomial extensions, the notions of a coherent sequence of polynomial ideals and of compatibility between polynomial and operator ideals were introduced in [18]. A variant of these notions that involves pairs formed by polynomial and multilinear ideals was considered in [27]. All these concepts are related to the concept of Property (B) and to holomorphy types (see [6]).

The search for such good nonlinear extensions was carried out with considering several different approaches (as the linearization and the factorization introduced by Pietsch in [32] or the more recent one introduced in [3], based on locally $\mathcal{I}$-bounded sets). In particular, the ideal of absolutely summing linear operators has been extended in a wide range of possibilities that have been faced by comparing which properties are satisfied in each class (see [16; 31]). A relevant polynomial ideal is the one formed by all p-dominated polynomials, which play a central role in the theory and have been intensively studied [7; 8; 9; 10; 12; 13; 24]. They were introduced as a generalization of absolutely p-summing linear operators, and their interest lies in the fact that they fulfill a Pietsch-type domination theorem. In fact, more than ten different generalizations of the original Pietsch domination theorem to nonlinear operator ideals have been obtained in the last 10–15 years (see [11; 19; 28; 30] and the references therein).

The theory of multilinear summing operators has found applications to quantum information theory. For example, very recently, estimates for the constants of the multilinear Bohnenblust–Hille inequality (case of real scalars) were used to

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