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Classification of Involutions on Enriques Surfaces

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Abstract. We present the classification of involutions on Enriques surfaces. We classify them into 18 types with the help of lattice theory due to Nikulin. We also give geometric realizations to all types.

1. Introduction

An Enriques surface Y is a compact complex surface satisfying the following conditions:

(1) the geometric genus and the irregularity vanish,
(2) the bi-canonical divisor on Y is linearly equivalent to 0.

Every Enriques surface Y is the quotient of a $K3$ surface X by a fixed point free involution ε. In this work, we give the classification of involutions on Enriques surfaces.

Given an involution ι on Y, we get two lifted involutions g and τ on X, which together with ε form an action by the Klein four-group $K_4 \simeq (\mathbb{Z}/2\mathbb{Z})^2$. Here g is the so-called symplectic or Nikulin involution, namely which acts on the space $H^0(X, \Omega^2)$ trivially. The other two act nonsymplectically. Conversely, if an action by K_4 on X contains a fixed point free involution ε, then the group $K_4/\langle\varepsilon\rangle$ determines an involution on $Y = X/\varepsilon$. Therefore our problem is equivalent to the classification of such K_4-actions.

By the Torelli theorem [PS], group actions on $K3$ surfaces are determined by the representation on the second cohomology group $H^2(X, \mathbb{Z})$, which has the natural structure of a unimodular lattice given by the cup product. To classify K_4-actions, we use the theory of classification of involutions of a lattice with condition on a sublattice, due to V. V. Nikulin [Nik4] (see Section 3 for a review and notation).

Let S be a fixed lattice and θ be an involution of S. In [Nik4], the determining condition of a triple (L, ϕ, i) with the condition (S, θ) satisfying the following commutative diagram is given.

$$\begin{array}{ccc} L & \xrightarrow{\ \phi\ } & L \\ {\scriptstyle i}\uparrow & & \uparrow{\scriptstyle i} \\ S & \xrightarrow[\ \theta\ ]{} & S \end{array}$$

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