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Generalized Maximum Principles and the Characterization of Linear Weingarten Hypersurfaces in Space Forms

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1. Introduction and Statement of the Main Results

Many authors have approached the problem of characterizing hypersurfaces immersed with constant mean curvature or with constant scalar curvature in a Riemannian space form $\mathbb{Q}_c^{n+1}$ of constant sectional curvature c. For instance, in the seminal work [8], Cheng and Yau introduced a new self-adjoint differential operator $\square$ acting on smooth functions defined on Riemannian manifolds. As a byproduct of such an approach, they were able to classify closed hypersurfaces M^n with constant normalized scalar curvature R satisfying $R \geq c$ and nonnegative sectional curvature immersed in $\mathbb{Q}_c^{n+1}$. Later on, Li [12] extended the results of Cheng and Yau [8] in terms of the squared norm of the second fundamental form of the hypersurface M^n. Shu [19] applied the generalized maximum principle of Omori [16] and Yau [21] to prove that a complete hypersurface M^n in the hyperbolic space $\mathbb{H}^{n+1}$ with constant normalized scalar curvature and nonnegative sectional curvature must be either totally umbilical or isometric to a hyperbolic cylinder $\mathbb{H}^1(-\sqrt{1+r^2}) \times \mathbb{S}^{n-1}(r)$ of $\mathbb{H}^{n+1}$. Brasil Jr., Colares, and Palmas [4] used the generalized maximum principle of Omori–Yau to characterize complete hypersurfaces with constant scalar curvature in $\mathbb{S}^{n+1}$. By applying a weak Omori–Yau maximum principle stated by Pigola, Rigoli, and Setti [17], Alías and García-Martínez [2] studied the behavior of the scalar curvature R of a complete hypersurface immersed with constant mean curvature into a real space form $\mathbb{Q}_c^{n+1}$, deriving a sharp estimate for the infimum of R. More recently, Alías, García-Martínez, and Rigoli [3] obtained another suitable weak maximum principle for complete hypersurfaces with constant scalar curvature in $\mathbb{Q}_c^{n+1}$ and gave some applications in order to estimate the norm of the traceless part of its second fundamental form. In particular, they extended the main theorem of [4] for the context of $\mathbb{Q}_c^{n+1}$.

Li [13] studied the rigidity of compact hypersurfaces with nonnegative sectional curvature immersed into a unit sphere with scalar curvature proportional to

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