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Domains with a Contracting Automorphism at a Boundary Point

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1. Introduction

The aim of this paper is to classify smoothly bounded pseudoconvex domains with a contracting automorphism at a boundary point. According to Kim–Yoccoz [10], this is the same as to study the smoothly bounded realization of weighted homogeneous models. Let us consider the complex Euclidean space $\mathbb{C}^{n+1}$ with the standard coordinates $(w, z) = (w, z_1, \dots, z_n)$. By a *weight* to the vector z, we mean an n-tuple $\delta = (\delta_1, \dots, \delta_n)$ of nonnegative real numbers. Given the weight δ, the *total degree* of the monomial $z^\alpha z^{\bar\beta} = z_1^{\alpha_1} \cdots z_n^{\alpha_n} \bar z_1^{\beta_1} \cdots \bar z_n^{\beta_n}$ of multi-indices $\alpha = (\alpha_1, \dots, \alpha_n)$ and $\beta = (\beta_1, \dots, \beta_n)$ is defined by $\delta(\alpha + \beta) = \sum_{j=1}^n \delta_j(\alpha_j + \beta_j)$. We say that a polynomial Q in z, $\bar z$ is *weighted homogeneous* if each monomial of Q has the same total degree for the given weight δ, that is, Q can be written as $Q(z, \bar z) = \sum_{\delta(\alpha+\beta)=\mu} Q_{\alpha\bar\beta} z^\alpha z^{\bar\beta}$ for some complex numbers $Q_{\alpha\bar\beta}$. A weighted homogeneous polynomial Q is said to be *balanced* if $Q_{\alpha\bar\beta} \neq 0$ only for (α, β) with $\delta(\alpha) = \delta(\beta)$.

A *weighted homogeneous model* is a domain in $\mathbb{C}^{n+1}$ defined by

$$M_P = \{(w, z) \in \mathbb{C} \times \mathbb{C}^n : \operatorname{Re} w + P(z, \bar z) < 0\}, \tag{1.1}$$

where P is a weighted homogeneous polynomial of total degree 1 for a weight $\delta = (\delta_1, \dots, \delta_n)$. Each weighted homogeneous model M_P admits the *dilation,*

$$\mathcal{D}_t(w, z) = (e^t w, e^{\delta_1 t} z_1, \dots, e^{\delta_n t} z_n) \quad (t \in \mathbb{R}),$$

and the *translation,*

$$\mathcal{T}_t(w, z) = (w + it, z) \quad (t \in \mathbb{R}),$$

as its automorphisms. Thus M_P has a noncompact automorphism group. Simultaneously, the dilation $\mathcal{D}_t$ with $t \neq 0$ extends to the CR automorphism of the boundary ∂M_P which is contracting or dilating at the origin.

In this paper, we shall prove the following theorem.

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