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Local Dynamics of Holomorphic Maps in $\mathbf{C}^2$ with a Jordan Fixed Point

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1. Introduction

Many authors have studied the local dynamics of holomorphic maps in $\mathbf{C}^n$ around a fixed point; see, for example, [4; 6] for an introduction to this field and known results. Most of the results are obtained under the assumption that the linear part of the map at the fixed point is diagonalizable. There are few results in the non-diagonalizable case. In [8], Coman and Dabija studied a special map with a Jordan fixed point and described its stable and unstable manifolds. In [1], Abate provided a systematic way of diagonalizing a map with a Jordan fixed point and proved several results under certain assumptions. In [2], Abate showed the existence of "parabolic curves" for holomorphic maps in $\mathbf{C}^2$ with an isolated Jordan fixed point. In [3], Abate studied a special map with a Jordan fixed point and proved the existence of an attracting domain under certain conditions. The aim of this paper is to provide a detailed study of the local dynamics of holomorphic maps in $\mathbf{C}^2$ with a Jordan fixed point.

Let f be a holomorphic map in $\mathbf{C}^2$ with a Jordan fixed point. In suitable local coordinates (z, w), f can be written as

$$\begin{aligned} z_1 &= \lambda(z + w + p_2(z, w) + p_3(z, w) + \cdots), \\ w_1 &= \lambda(w + q_2(z, w) + q_3(z, w) + \cdots) \end{aligned} \tag{1.1}$$

if $\lambda \neq 0$ and as

$$\begin{aligned} z_1 &= w + p_2(z, w) + p_3(z, w) + \cdots, \\ w_1 &= q_2(z, w) + q_3(z, w) + \cdots \end{aligned} \tag{1.2}$$

if $\lambda = 0$. Here $p_i(z, w)$ and $q_i(z, w)$ are homogeneous polynomials of degree i.

If $|\lambda| \neq 1$ then we say that f has a *hyperbolic* Jordan fixed point. If $|\lambda| = 1$ and λ is not a root of unity, then we say that f has an *elliptic* Jordan fixed point. If λ is a root of unity, then we say that f has a *parabolic* Jordan fixed point and we can consider a suitable iteration of f instead. Thus we will assume that $\lambda = 1$ in the parabolic case.

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