Michigan Math. J. 62 (2013), 753–768

On Computations of Genus 0 Two-Point Descendant Gromov–Witten Invariants

Amin Gholampour & Hsian-Hua Tseng

1. Introduction

Let X be a smooth proper Deligne–Mumford $\mathbb{C}$-stack with projective coarse moduli space. Genus 0 two-point descendant Gromov–Witten invariants of X are invariants of the following kind:

$$\langle a\psi^k, b\psi^l\rangle_{0,2,\beta}^X := \int_{[\overline{\mathcal{M}}_{0,2}(X,\beta)]^{\mathrm{vir}}} \mathrm{ev}_1^*(a)\psi_1^k\, \mathrm{ev}_2^*(b)\psi_2^l, \tag{1.1}$$

where $a, b \in H^*(IX)$, $k, l \in \mathbb{Z}_{\geq 0}$, and $\mathrm{ev}_1, \mathrm{ev}_2 \colon \overline{\mathcal{M}}_{0,2}(X,\beta) \to IX$ are the evaluation maps. We refer to [1] for the basics of the construction of Gromov–Witten invariants for Deligne–Mumford stacks.

Recently, exact computations of genus 0 two-point descendant Gromov–Witten invariants have received much attention because of mirror symmetry for genus 1 and open Gromov–Witten invariants. In the case $X = \mathbb{P}^n$, a formula for the invariants (1.1) is proved in [14]. Formulas for variants of (1.1) involving twists by Euler class and direct sums of line bundles, in the sense of [4], are also proven in [14] and [12] in the toric setting. More recently, a formula for the invariants (1.1) for compact symplectic toric manifolds is proven in [11]. The proofs in [11; 12; 14] follow a strategy that is similar to the one used by Givental in his computation of genus 0 one-point descendant invariants [5; 6]. More precisely, a generating function of invariants (1.1) is proven by virtual localization to satisfy certain recursion relations and certain regularity conditions. The localization computations needed in [11; 12; 14] are somewhat involved.

The purpose of this paper is to discuss a simpler method for explicitly computing (1.1). This method is based on a known fact in topological field theory that relates two-point descendant invariants (1.1) to one-point descendant invariants; see equation (2.5). We explain this method in detail in Section 2. In Section 3 we apply this method to compute two-point descendant invariants for several classes of examples.

Convention. We work over the field of complex numbers. Cohomology groups are taken with rational coefficients. In this paper we consider cohomology only in even degrees.

Received August 27, 2012. Revision received May 3, 2013.