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On the Representation of Holomorphic Functions on Polyhedra

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1. Introduction

1.1. Oka's Theorem

The following beautiful theorem of Oka, which gives a representation for holomorphic functions defined on p-polyhedra in $\mathbb{C}^d$, has played a significant role in the development of several complex variables.

THEOREM 1.1 (Oka [26], as presented in [7]). *Let $\delta_1, \dots, \delta_m$ be a collection of polynomials in d variables normalized such that the p-polyhedron K_δ defined by*

$$K_\delta = \{\lambda \in \mathbb{C}^d \mid |\delta_l(\lambda)| \le 1 \text{ for } l = 1, \dots, m\}$$

lies in $\mathbb{D}^d$. If ϕ is holomorphic on a neighborhood of K_δ, then there exists a function Φ, holomorphic on a neighborhood of $(\mathbb{D}^-)^{d+m}$, such that

$$\phi(\lambda) = \Phi(\lambda, \delta(\lambda))$$

for all $\lambda \in K_\delta$.

Introduced originally in 1936 to give an elegant new proof of the Oka–Weil approximation theorem [26; 33], Oka's theorem was a stem theorem for the development of the theory of analytic sheaves—a powerful tool for applying function theory to domains of holomorphy and, more generally, Stein spaces [19; 21]. Basic to the understanding of polynomial convexity, Oka's theorem played an important role in the development of the theory of Banach algebras. Many operator theorists first learn of this theorem in the context of one of its many basic implications: the Arens–Calderon trick [10], which is fundamental to spectral theory and to the corresponding functional calculus for commuting tuples of operators [18; 28; 29].

1.2. Oka Mappings

In this paper we show how ideas that are currently evolving within the operator theory community can be adapted to obtain precise bounds for Oka's theorem.

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