Michigan Math. J. 62 (2013), 537–550

On Symplectic Automorphisms of Hyper-Kähler Fourfolds of $K3^{[2]}$ Type

Giovanni Mongardi

1. Introduction

An automorphism φ of a hyper-Kähler manifold X is *symplectic* if

$$\varphi^*(\sigma_X) = \sigma_X,$$

where σ_X is a holomorphic symplectic 2-form on X. Finite abelian groups of symplectic automorphisms of complex K3 surfaces have been classified by Nikulin in [11]. In particular, we know that a symplectic automorphism of finite order on a K3 surface over $\mathbb{C}$ has order at most 8.

This paper deals with symplectic automorphisms on hyper-Kähler fourfolds that are deformation equivalent to the Hilbert scheme of two points of a K3 surface; such fourfolds are known as manifolds of $K3^{[2]}$ type. Recall that manifolds of $K3^{[2]}$ type have $b_2 = 23$ and $H^2_{\mathbb{Z}} \cong U^3 \oplus E_8(-1)^2 \oplus (-2)$; here U is the hyperbolic plane, $E_8(-1)$ is the unique negative-definite even unimodular lattice of rank 8, and (-2) is the rank-1 lattice of discriminant -2.

Let Co_1 be Conway's sporadic simple group. The main result of this paper is the following theorem.

Theorem 1.1. *Let X be a hyper-Kähler manifold of $K3^{[2]}$ type and let G be a finite group of symplectic automorphisms of X. Then G is isomorphic to a subgroup of* Co_1.

We recall that Mukai [10] proved an analogous result for K3 surfaces (see also the proof of Kondo [8]). Namely, a finite group of symplectic automorphisms is a subgroup of Mathieu's group M_{23}.

A partial converse to Theorem 1.1 is provided by Proposition 2.12, which also gives a computational method of determining possible finite automorphism groups. We shall use Theorem 1.1 to prove the following result on symplectic automorphisms of order 11.

Proposition 1.2. *Let X be a fourfold of $K3^{[2]}$ type and let $\psi: X \to X$ be a symplectic automorphism of order 11. Then* $21 \geq h^{1,1}_{\mathbb{Z}}(X) \geq 20$. *Moreover,* $\mathrm{Bir}(X)$

Received May 15, 2012. Revision received February 4, 2013.