Michigan Math. J. 62 (2013), 143–184

Double Covers of EPW-Sextics

Kieran G. O'Grady

0. Introduction

EPW-sextics are defined as follows. Let V be a 6-dimensional complex vector space. Choose a volume form $\mathrm{vol}\colon \bigwedge^6 V \xrightarrow{\sim} \mathbb{C}$ and equip $\bigwedge^3 V$ with the symplectic form

$$(\alpha,\beta)_V := \mathrm{vol}(\alpha\wedge\beta). \tag{0.0.1}$$

Let $\mathbb{LG}\big(\bigwedge^3 V\big)$ be the symplectic Grassmannian parameterizing Lagrangian subspaces of $\bigwedge^3 V$; of course, $\mathbb{LG}\big(\bigwedge^3 V\big)$ does not depend on the choice of volume form. Let $F \subset \bigwedge^3 V \otimes \mathcal{O}_{\mathbb{P}(V)}$ be the subvector bundle with fiber

$$F_v := \Big\{\alpha\in \bigwedge^3 V \mid v\wedge\alpha = 0\Big\} \tag{0.0.2}$$

over $[v]\in\mathbb{P}(V)$. Observe that $(\cdot,\cdot)_V$ is zero on F_v and that $2\dim(F_v) = 20 = \dim \bigwedge^3 V$; hence F is a Lagrangian subvector bundle of the trivial symplectic vector bundle on $\mathbb{P}(V)$ with fiber $\bigwedge^3 V$. Next choose $A\in\mathbb{LG}\big(\bigwedge^3 V\big)$. Let

$$F \xrightarrow{\lambda_A} \Big(\bigwedge^3 V/A\Big)\otimes\mathcal{O}_{\mathbb{P}(V)} \tag{0.0.3}$$

be the composition of the inclusion $F\subset \bigwedge^3 V\otimes\mathcal{O}_{\mathbb{P}(V)}$ followed by the quotient map. Since $\mathrm{rk}\,F = \dim(V/A)$, the determinant of λ_A makes sense. Let

$$Y_A := V(\det\lambda_A).$$

A straightforward computation gives that $\det F\cong\mathcal{O}_{\mathbb{P}(V)}(-6)$ and hence $\det\lambda_A\in H^0(\mathcal{O}_{\mathbb{P}(V)}(6))$. It follows that if $\det\lambda_A\neq 0$ then Y_A is a sextic hypersurface. As is easily checked, $\det\lambda_A\neq 0$ for generic $A\in\mathbb{LG}\big(\bigwedge^3 V\big)$ (note that there exist "pathological" As such that $\lambda_A = 0$; e.g., $A = F_{v_0}$). An *EPW-sextic* (after Eisenbud, Popescu, and Walter [5]) is a sextic hypersurface in $\mathbb{P}^5$ that is projectively equivalent to Y_A for some $A\in\mathbb{LG}\big(\bigwedge^3 V\big)$. Let Y_A be an EPW-sextic. One can construct a coherent sheaf ξ_A on Y_A and a multiplication map $\xi_A\times\xi_A\to\mathcal{O}_{Y_A}$ that gives $\mathcal{O}_{Y_A}\oplus\xi_A$ the structure of an $\mathcal{O}_{Y_A}$-algebra; this is known to experts (see [3]), and we will give the construction in Section 1.2. The *double EPW-sextic* associated to A is $X_A := \mathrm{Spec}(\mathcal{O}_{Y_A}\oplus\xi_A)$; we let $f_A\colon X_A\to Y_A$ be the structure morphism. In [12] we considered X_A for generic A and proved that it is a hyper-Kähler deformation of $(\mathrm{K3})^{[2]}$ (the blow-up of the diagonal in the symmetric square of a K3

Received December 19, 2011. Revision received September 27, 2012.
The author was supported by PRIN 2007.