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Open Gromov–Witten Theory and the Crepant Resolution Conjecture

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1. Introduction

1.1. Summary of Results

Gromov–Witten invariants are virtual counts of curves on a fixed target space X, and they are obtained as intersection numbers on moduli spaces of Kontsevich-stable maps to X. Besides being interesting symplectic invariants of X, they exhibit a remarkable amount of algebraic structure: appropriate generating functions of rational Gromov–Witten invariants give a deformation of the intersection ring of X (quantum cohomology), also endowing the cohomology of X with the structure of a Frobenius manifold. There are two typical "families of questions" in Gromov–Witten theory. First, a simple-minded yet difficult question is whether it is possible to compute invariants for a given target. Second, one wishes to draw interesting consequences from the algebraic structure of invariants, such as comparing in some precise way families of invariants of different but related targets or relating different types of curve-counting invariants on the same target.

In this paper we combine these two kinds of questions to investigate two striking (conjectural) features of Gromov–Witten theory.

> *Crepant transformation:* the equivalence between GW theories of two targets related by a crepant birational transformation. In particular, when the crepant transformation is the resolution of singularities of a Gorenstein orbifold, this equivalence is referred to as the *crepant resolution conjecture* (CRC).
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> *Gluing:* the ability to recover GW invariants for a toric variety/orbifold from open invariants of open subspaces covering the target.

We seek to tackle such questions for arbitrary toric spaces (varieties or orbifolds) by reducing them to local questions that are compatible with gluing procedures. We provide an expanded discussion of our motivations in Section 1.2. Here we present the specific results obtained in this paper.

We give a complete and exhaustive description for the specific geometry in Figure 1. The global quotient $\mathcal{X} = [\mathcal{O}_{\mathbb{P}^1}(-1) \oplus \mathcal{O}_{\mathbb{P}^1}(-1)/\mathbb{Z}_2]$ (with nontrivial

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