Michigan Math. J. 61 (2012), 675–701

Residual Intersections of Licci Ideals Are Glicci

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Dedicated to Mel Hochster for his groundbreaking work in commutative algebra

1. Introduction

This paper deals with the interplay of two generalizations of classical complete intersection linkage, namely Gorenstein linkage and residual intersection. Recall that two proper ideals I and J in a Cohen–Macaulay ring R are said to be *linked* or *linked with respect to* $\mathfrak{a}$, written $I \sim J$, if $J = \mathfrak{a} : I$ and $I = \mathfrak{a} : J$ for some complete intersection ideal $\mathfrak{a}$. The link is called *geometric* in case $I + J$ has height at least $g + 1$, where $g = \operatorname{ht} \mathfrak{a}$. As is well known, two linked ideals are automatically unmixed of height g, and conversely, whenever $\mathfrak{a} \subsetneq I$ are ideals of height g in a Gorenstein ring with $\mathfrak{a}$ a complete intersection and I unmixed, then $I \sim \mathfrak{a} : I$ is a link [32] (see also [15]). A sequence of two links $I \sim J \sim K$ is referred to as a *double link*. In a similar manner we define the (*even, odd*) *linkage class* of an ideal I, which is the set of all ideals obtained from I by a finite (even, odd) number of links. Finally, one says that an ideal is *licci* if it belongs to the *li*nkage *c*lass of a *c*omplete *i*ntersection. Licci ideals have been studied extensively. They are known to be perfect for instance, and hence define Cohen–Macaulay rings [32]. In addition they share more subtle homological properties of complete intersections: Licci ideals are strongly nonobstructed (provided R is Gorenstein) [5], are strongly Cohen–Macaulay [21], and the shifts in their homogeneous minimal free resolution grow "sufficiently fast" (provided R is a polynomial ring over a field and the ideal is homogeneous) [24].

These more refined properties have often been used to verify that a given ideal fails to be licci, but they also suggest that the classification provided by complete intersection linkage might be too fine for some purposes. Hence in recent years the emphasis has shifted to the more inclusive notion of *Gorenstein linkage,* where the complete intersection ideal $\mathfrak{a}$ in the definition of linkage is replaced by an unmixed Gorenstein ideal $\mathfrak{a}$, meaning an unmixed ideal such that $R/\mathfrak{a}$ is Gorenstein. The first systematic study of this notion can be found in [34]. Again one talks about *Gorenstein double linkage,* the (*even, odd*) *Gorenstein linkage class* of an ideal, and the property of being *glicci,* which means that an ideal belongs to the *G*orenstein *li*nkage *c*lass of a *c*omplete *i*ntersection. The same remarks as above apply to Gorenstein linkage, except that glicci ideals no longer have the more

Received May 9, 2011. Revision received August 4, 2011.
The second and third author gratefully acknowledge support from NSF Grant no. DMS-0756853 and DMS-0901613, respectively.