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Enriques Surfaces: Brauer Groups and Kummer Structures

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1. Introduction

The Brauer group is an important but very subtle birational invariant of a projective surface. In [3], Beauville proved that generically the Brauer group of a complex Enriques surface injects into the Brauer group of the covering K3 surface. Subsequently Beauville asked for explicit examples where the Brauer groups pull back identically to zero. This problem has recently been solved in [6] (see also Section 7.10) and in [7], but only by isolated so-called singular K3 surfaces (Picard number 20). In this paper we develop methods to derive such surfaces in 1-dimensional families. Our results cover both the Kummer and the non-Kummer case. In Section 3, we construct for any integer $N > 1$ a 1-dimensional family $\mathcal{X}_N$ of complex K3 surfaces with Picard number $\rho \geq 19$ such that the general member admits an Enriques involution τ.

Theorem 1. *Let $N > 1$. Consider a general K3 surface $X_N \in \mathcal{X}_N$; that is, $\rho(X_N) = 19$. Denote the quotient by the Enriques involution τ by Z_N. Then*

$$\pi^* \operatorname{Br}(Z_N) = \begin{cases} \{0\} & \text{if } N \text{ is odd,} \\ \mathbb{Z}/2\mathbb{Z} & \text{if } N \text{ is even.} \end{cases}$$

The K3 surfaces in the family $\mathcal{X}_N$ are generally not Kummer, but in Section 5 we exploit a geometric construction related to Kummer surfaces of N-isogenous elliptic curves. By similar methods, in Section 7 we derive for any $N \in \mathbb{N}$ a 1-dimensional family $\mathcal{Y}_N$ consisting of Kummer surfaces with Picard number $\rho \geq 19$ and Enriques involution τ.

Theorem 2. *Let $N \in \mathbb{N}$. Consider a Kummer surface $Y_N \in \mathcal{Y}_N$ with $\rho(Y_N) = 19$. Let τ denote the Enriques involution on Y_N. Then*

$$\pi^* \operatorname{Br}(Y_N/\tau) = \begin{cases} \{0\} & \text{if } N \text{ is odd,} \\ \mathbb{Z}/2\mathbb{Z} & \text{if } N \text{ is even.} \end{cases}$$

The two theorems show that the Enriques surfaces in question come in families. We shall work out one family in detail in Theorem 14. The general assumption that the K3 surfaces have nonmaximal Picard number $\rho = 19$ is fairly mild and not strictly necessary (see Proposition 15 and Section 5.9).

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