1. Introduction

We work over an algebraically closed ground field $k$ of characteristic 0. If $G$ is a finite group then, by [8], a $G$-torsor $f : X \to Y$ in the category of algebraic varieties can be viewed as a tensor functor $\text{Rep}-G \to \text{Vect}(Y)$. More concretely, the associated tensor functor sends the representation $V$ to the vector bundle $f^*(V \otimes \mathcal{O})^G$. When the cover ramifies, as was observed in [9], we need to put tensor functors in the category of vector bundles with appropriate parabolic structure.

In the case where $Y = \mathbb{P}^1$ we have $f^*(V \times \mathcal{O})^G = \bigoplus \mathcal{O}(s_i)$. The integers $s_i$ are difficult to compute, and one of our results is to find an upper bound on them when there is ramification at 0, 1, and $\infty$ only. The bound described in Theorem 8.4 and Example 8.6 improves the known bound in [3]. There is one case in which it is easy to compute the integers $s_i$—namely, when the group $G$ is cyclic.

Our method is a type of reduction to the cyclic case by removing ramification at 0. More precisely, the endomorphism $z \mapsto z^n$ of $\mathbb{P}^1$ algebraically de-loops loops around the origin. Pulling back a cover along this morphism removes ramification of order $n$ at the origin. For our method to work we must define a pullback morphism for parabolic bundles. As in [6] and [3], this entails using the equivalence of categories (due to Biswas [2]) between parabolic bundles of a certain kind and vector bundles on an associated root stack. The pullback operation is difficult to reverse—that is, given a morphism $f : X \to Y$ of smooth projective curves and a parabolic bundle $\mathcal{F}$ on $X$, to construct a parabolic bundle on $Y$ that pulls back to $\mathcal{F}$. In fact, the difficulty in reversing the parabolic pullback gives a new explanation for why it is difficult to compute the $s_i$.

The interest in computing these $s_i$ can be explained as follows. A finite quotient $q : F_2 \to G$ of the free group on two letters produces a cover $X_q \to \mathbb{P}^1$ ramified at three points. The absolute Galois group $G_\mathbb{Q}$ of $\mathbb{Q}$ acts faithfully on such covers. For a given $q$, however, the Galois action is difficult to understand; and it is not known what finite quotient of $G_\mathbb{Q}$ acts in sending the cover to some other nonisomorphic cover. One way of addressing this question is to give a more algebraic construction of the cover. The theory of tannakian categories allows one to do this. One should view the cover as a tensor functor into parabolic bundles and then understand the Galois action on such tensor functors. This work should be