Michigan Math. J. 61 (2012), 133–150

Geodesic Continued Fractions

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1. Introduction

Every rational number can be expressed uniquely in the form p/q, where p and q are coprime integers and q is positive; we describe such rationals as *reduced*. Two reduced rationals p/q and r/s are *Farey neighbors* if $|ps-qr| = 1$. As usual, we adjoin the point ∞ to the set $\mathbb{Q}$ of rationals to form $\mathbb{Q}_\infty$. We then define $1/0$ to be the reduced form of ∞, and p/q to be a Farey neighbor of ∞ if and only if $|p.0 - q.1| = 1$ (i.e., if and only if p/q is an integer). The *Farey graph* $\mathcal{F}$ is the graph whose set of vertices is $\mathbb{Q}_\infty$ and whose edges join each pair of Farey neighbors (and only these). We denote the path in $\mathcal{F}$ that passes through the vertices $v_1, \dots, v_n$ in this order by $\langle v_1, \dots, v_n\rangle$. A concrete realization of $\mathcal{F}$ is obtained by joining each pair of Farey neighbors by a hyperbolic line in the upper half-plane model $\mathbb{H}$ of the hyperbolic plane. It is well known that any two such hyperbolic lines have at most an endpoint in common, and this set of hyperbolic lines induces the *Farey tessellation* of $\mathbb{H}$ into mutually disjoint, nonoverlapping, ideal hyperbolic triangles (see e.g. [7; 8; 15]). Henceforth $\mathcal{F}$ refers to this model of the Farey graph, which is illustrated in Figure 1.

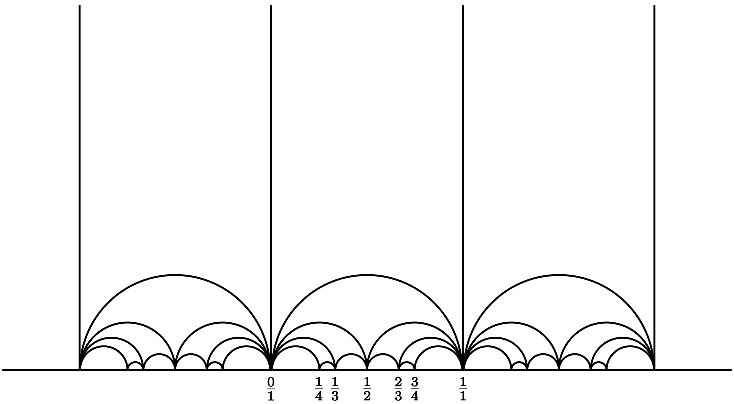

Figure 1 The Farey graph

Received July 2, 2010. Revision received September 28, 2011.
Part of the third author's work was carried out during an LMS–EPSRC Durham Symposium in 2011.