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Two Extension Theorems of Hartogs–Chirka Type Involving Continuous Multifunctions

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1. Introduction and Statement of Results

This paper is motivated (i) by a version of Hartogs' lemma stating that, if Ω is some neighborhood of the union of $\partial\mathbb{D} \times \mathbb{D}$ and a complex analytic subvariety $\Sigma \subset \bar{\mathbb{D}} \times \mathbb{D}$ that is finitely sheeted over $\mathbb{D}$ (such that $\Omega \cap \mathbb{D}^2$ is connected) and if $f \in \mathcal{O}(\Omega)$, then f continues holomorphically to $\mathbb{D}^2$ and (ii) by the Hartogs-type extension theorem of Chirka. Chirka's theorem reads as follows.

Result 1.1 (Chirka). *Let $\phi\colon \bar{\mathbb{D}} \to \mathbb{C}$ be a continuous function having $\sup_{z\in\bar{\mathbb{D}}}|\phi(z)| < 1$ and let S be its graph. Let Ω be a connected open neighborhood of $S \cup (\partial\mathbb{D} \times \mathbb{D})$ contained in $\{(z,w) \in \mathbb{C}^2 : |w| < 1\}$. If $f \in \mathcal{O}(\Omega)$, then f extends holomorphically to $\mathbb{D}^2$.*

(Here and in what follows, $\mathbb{D}$ denotes the open unit disc in $\mathbb{C}$.) This motivates the question of whether—given the "Weierstrass pseudopolynomial"

$$\mathcal{P}_a(z,w) := w^k + \sum_{j=0}^{k-1} a_j(z)w^j, \quad k \geq 2 \tag{1.1}$$

(where $a_0,\dots,a_{k-1} \in \mathcal{C}(\bar{\mathbb{D}})$ with $\mathcal{P}_a^{-1}\{0\} \subset \bar{\mathbb{D}} \times \mathbb{D}$) and given a neighborhood Ω of $\mathcal{P}_a^{-1}\{0\} \cup (\partial\mathbb{D} \times \bar{\mathbb{D}})$—the aforementioned results hold in this new setting.

One possible approach to this question is to investigate a version of Result 1.1 with one copy of $\mathbb{D}$ replaced by a bordered Riemann surface determined by $a := (a_0,\dots,a_{k-1})$, over which the graph of the multifunction is transformed to a graph. One is then reduced to solving a certain quasilinear $\bar{\partial}$-problem analogous to the one considered by Chirka in [4] (also see [5]). There is considerable literature on this subject (see, e.g., [7]). However, for this approach to work, one needs continuous dependence of solutions on the parameters as well as sup-norm estimates with small norm, neither of which seem to be known at this time. A second approach is suggested by the Kontinuitätssatz-based strategies of Bharali [2] and Barrett-Bharali [1], provided one is willing to allow $(a_0,\dots,a_{k-1})$ in (1.1) to belong to some strict subclass of $\mathcal{C}(\bar{\mathbb{D}};\mathbb{C}^k)$. To motivate the origins of the two main theorems that follow, we state one of the results from [1] and [2].

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