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# Moduli Spaces of Rank-2 ACM Bundles on Prime Fano Threefolds

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## 1. Introduction

A vector bundle $F$ on a smooth polarized variety $(X, H_X)$ has no intermediate cohomology if $\mathrm{H}^k(X, F \otimes \mathcal{O}_X(tH_X)) = 0$ for all $t \in \mathbb{Z}$ and $0 < k < \dim(X)$. These bundles are also called *arithmetically Cohen–Macaulay* (*ACM*) because they correspond to maximal Cohen–Macaulay modules over the coordinate ring of $X$. It is known that an ACM bundle must be a direct sum of line bundles if $X = \mathbb{P}^n$ [39] or a direct sum of line bundles and (twisted) spinor bundles if $X$ is a smooth quadric hypersurface in $\mathbb{P}^n$ [53; 72]. On the other hand, there exists a complete classification of varieties admitting, up to twist, a finite number of isomorphism classes of indecomposable ACM bundles [16; 25]. Only five cases exist besides rational normal curves, projective spaces, and quadrics.

For varieties that are not on this list, the problem of classifying ACM bundles has been taken up only in some special cases. For instance, on general hypersurfaces in $\mathbb{P}^n$ of dimension at least 3, a full classification of ACM bundles of rank 2 is available; see [22; 23; 56; 63; 64]. For dimension 2 and rank 2, a partial classification can be found in [12; 20; 21; 27]. For higher rank, some results are given in [7; 19].

The case of smooth Fano threefolds $X$ with Picard number 1 has also been studied. In this case one has $\mathrm{Pic}(X) \cong \langle H_X \rangle$, with $H_X$ ample, and the canonical divisor class $K_X$ satisfies $K_X = -i_X H_X$, where the *index* $i_X$ satisfies $1 \le i_X \le 4$. Recall that $i_X = 4$ implies $X \cong \mathbb{P}^3$ and $i_X = 3$ implies that $X$ is isomorphic to a smooth quadric. Thus, the class of ACM bundles is completely understood in these two cases.

In contrast to this, the cases $i_X = 2, 1$ are highly nontrivial. First of all, there are several deformation classes of these varieties [49; 51; 52]. A different approach to the classification of these varieties was proposed by Mukai [67; 68; 69].

In the second place, it is still unclear how to characterize the invariants of ACM bundles; in fact, the investigation has been thoroughly carried out only in the case of rank 2. For $i_X = 2$, the classification was completed in [5]. For $i_X = 1$, a result of Madonna [57] implies that if a rank-2 ACM bundle $F$ is defined on $X$, then its

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