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Weighted C^k Estimates for a Class of Integral Operators on Nonsmooth Domains

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1. Introduction

Let X be an n-dimensional complex manifold equipped with a Hermitian metric, and let $D \subset\subset X$ be a strictly pseudoconvex domain with defining function r. Here we do not assume the nonvanishing of the gradient, dr, thus allowing for the possibility of singularities in the boundary, ∂D, of D. We refer to such domains as Henkin–Leiterer domains, as they were first systematically studied by Henkin and Leiterer in [2].

We shall make the additional assumption that r is a Morse function.

Let $\gamma = |\partial r|$. In [1] the author established an integral representation of the following form.

Theorem 1.1. *There exist integral operators* $\tilde{T}_q : L^2_{(0,q+1)}(D) \to L^2_{(0,q)}(D)$ *with* $0 \le q < n = \dim X$ *such that, for* $f \in L^2_{(0,q)} \cap \operatorname{Dom}(\bar\partial) \cap \operatorname{Dom}(\bar\partial^*)$, *one has*

$$\gamma^3 f = \tilde{T}_q \bar\partial f + \tilde{T}^*_{q-1} \bar\partial^* f + (\textit{error terms}) \quad \textit{for } q \ge 1.$$

Theorem 1.1 is valid under the assumption that we are working with the Levi metric. With local coordinates denoted by $\zeta_1, \dots, \zeta_n$, we define a Levi metric in a neighborhood of ∂D by

$$ds^2 = \sum_{j,k} \frac{\partial^2 r}{\partial \zeta_j, \partial \bar\zeta_k}(\zeta).$$

A Levi metric on X is a Hermitian metric that is a Levi metric in a neighborhood of ∂D. In what follows we will be working with X equipped with a Levi metric.

The author [1] then used properties of the operators in the representation to establish the following estimates.

Theorem 1.2. *For* $f \in L^2_{0,q}(D) \cap \operatorname{Dom}(\bar\partial) \cap \operatorname{Dom}(\bar\partial^*)$ *with* $q \ge 1$,

$$\|\gamma^{3(n+1)} f\|_{L^\infty} \lesssim \|\gamma^2 \bar\partial f\|_\infty + \|\gamma^2 \bar\partial^* f\|_\infty + \|f\|_2.$$

In this paper we examine the operators in the integral representation, derive more detailed properties of such operators under differentiation, and use the properties to establish C^k estimates. Our main theorem is as follows.

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