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On Moduli Spaces of Parabolic Vector Bundles of Rank 2 over $\mathbb{CP}^1$

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1. Introduction

Let $S \subset \mathbb{CP}^1$ be a finite subset such that $\#S \geq 5$. Fix an integer d. Let $\mathcal{M}_S(d) = \mathcal{M}_S$ be the moduli space of parabolic semistable vector bundles $E_* \to \mathbb{CP}^1$ of rank 2 and degree d with parabolic structure over S such that for each point $s \in S$ the parabolic weights of E_* at s are 0 and $1/2$. In [4], geometric realizations of the variety $\mathcal{M}_S$ were obtained by the third author (under the assumption that $\#S$ is even).

Our aim here is to address the following Torelli type question:

> Take two subsets S_1 and S_2 such that the variety $\mathcal{M}_{S_1}$ is isomorphic to $\mathcal{M}_{S_2}$. Does this imply that the multi-pointed curve $(\mathbb{CP}^1, S_1)$ is isomorphic to $(\mathbb{CP}^1, S_2)$?

The following theorem proved here (see Theorem 4.2) shows that this indeed is the case.

Theorem 1.1. *Take two finite subsets S_1 and S_2 of $\mathbb{CP}^1$ of cardinality ≥ 5. The variety $\mathcal{M}_{S_1}$ is isomorphic to $\mathcal{M}_{S_2}$ if and only if there is an automorphism*

$$\varphi\colon \mathbb{CP}^1 \to \mathbb{CP}^1$$

such that $\varphi(S_1) = S_2$.

If $\#S = 4$, then the moduli space $\mathcal{M}_S$ is isomorphic to $\mathbb{CP}^1$. Therefore, the assumption in Theorem 1.1 that there are at least five parabolic points is necessary.

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2. Hitchin Map and Unstable Locus

Let

$$S \subset \mathbb{CP}^1$$

be a finite subset of the complex projective line such that

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