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Scalar Curvature Behavior for Finite-Time Singularity of Kähler–Ricci Flow

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1. Introduction

Ricci flow, since its debut in the famous original work [4] by Hamilton, has been one of the major driving forces for the development of geometric analysis in the past decades. Its astonishing power is best demonstrated by the breakthrough in solving the Poincaré conjecture and geometrization program. For this amazing story, we refer to [1; 7; 10] and the references therein. Meanwhile, Kähler–Ricci flow, which is Ricci flow with initial metric being Kähler, has shown some of its own characters coming from the natural relation with complex Monge–Ampère equation and many interesting algebraic geometric objects. Tian's program, as described in [14] or [15], has illustrated the direction to further improve people's understanding in many classic topics of great importance by Kähler–Ricci flow—for example, the minimal model program in algebraic geometry.

In this paper we give a very general discussion on Kähler–Ricci flows over closed manifolds. The closed manifold under consideration is denoted by X with $\dim_{\mathbb{C}} X = n$. The computation would be done for the following version of Kähler–Ricci flow:

$$\frac{\partial \tilde{\omega}_t}{\partial t} = -\mathrm{Ric}(\tilde{\omega}_t) - \tilde{\omega}_t, \quad \tilde{\omega}_0 = \omega_0, \tag{1.1}$$

where ω_0 is any Kähler metric on X. The special feature of this version as shown in [15] and fully discussed in [17] is rather superficial for this work.

The short time existence of the flow is known from either Hamilton's general existence result on Ricci flow in [4] or the fact that Kähler–Ricci flow is indeed parabolic when considered as a flow in a properly chosen infinite-dimensional space.

In light of the optimal existence result for Kähler–Ricci flow as in [2] or [15], we know the classic solution of (1.1) exists exactly as long as the cohomology class $[\tilde{\omega}_t]$ from formal computation remains Kähler. The actual meaning will be explained later.

Inevitably, it comes down to analyzing the behavior of the t-slice metric solution when time t approaches the (possibly infinite) singular time from cohomology consideration. In this work, we focus on the case when the flow singularity happens at some finite time. Now we state the main results.

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