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Spectral Characteristics and Stable Ranks for the Sarason Algebra $H^\infty + C$

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0. Introduction

We prove a corona-type theorem with bounds for the Sarason algebra $H^\infty + C$ and determine its spectral characteristics, thus continuing a line of research initiated by N. Nikolski. We also determine the Bass, the dense, and the topological stable ranks of $H^\infty + C$.

To fix our setting, let A be a commutative unital Banach algebra with unit e and let $M(A)$ be its maximal ideal space. The following concept of spectral characteristics was introduced by Nikolski [15]. For $a \in A$, let $\hat{a}$ denote the Gelfand transform of a. We let

$$\delta(a) = \min_{t \in M(A)} |\hat{a}(t)|.$$

Note that $\delta(a) \le \|\hat{a}\|_\infty \le \|a\|_A$. When $a = (a_1, \dots, a_n) \in A^n$ we define

$$\delta_n(a) = \min_{t \in M(A)} |\hat{a}(t)|,$$

where $|\hat{a}(t)| = \sum_{j=1}^n |\hat{a}_j(t)|$ for $t \in M(A)$, and we let

$$\|a\|_{A^n} = \max\{\|a_1\|_A, \dots, \|a_n\|_A\}.$$

Typically, one defines $|\hat{a}(t)| = |\hat{a}(t)|_2 := \big(\sum_{j=1}^n |\hat{a}_j(t)|^2\big)^{1/2}$ and $\|a\|_{A^n} = \|a\|_2 := \big(\sum_{j=1}^n \|a_j\|_A^2\big)^{1/2}$. Our later calculations will be easier, though, with the present definition.

Let δ be a real number satisfying $0 < \delta \le 1$. We are interested in finding, or bounding, the functions

$$c_1(\delta, A) = \sup\{\|a^{-1}\|_A : \|a\|_A \le 1,\ \delta(a) \ge \delta\}$$

and

$$c_n(\delta, A) = \sup\left\{\inf\left\{\|b\|_{A^n} : \sum_{j=1}^n a_j b_j = e\right\},\ \|a\|_{A^n} \le 1,\ \delta_n(a) \ge \delta\right\} \tag{0.1}$$

when A is the Sarason algebra $H^\infty + C$. If a is not invertible, we define $\|a^{-1}\| = \infty$.

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