Michigan Math. J. 59 (2010), 329–352

2D Inviscid Heat Conductive Boussinesq Equations on a Bounded Domain

Kun Zhao

1. Introduction

One major challenge in fluid dynamics is the question of global existence and large-time asymptotic behavior of solutions to certain initial value (Cauchy) problems or initial-boundary value problems (IBVP) for modeling equations. For decades, the question of global existence/finite time blow-up of smooth solutions for the three-dimensional incompressible Euler or Navier–Stokes equations has been one of the most outstanding open problems in applied analysis. The answer to this question will play an important role in understanding core problems in fluid dynamics such as the onset of turbulence. Enormous efforts have been made on this subject, but the resolution of some basic issues is still missing. The main difficulty is to understand the vortex stretching effect in 3D flows. As part of the effort to understand the vortex stretching effect in 3D flows, various simplified model equations have been proposed. Among these models, the 2D Boussinesq system is known to be one of the most commonly used because it is analogous to the 3D incompressible Euler or Navier–Stokes equations for axisymmetric swirling flow, and it shares a similar vortex stretching effect as that in the 3D incompressible flow. Better understanding of the 2D Boussinesq system will undoubtedly shed light on the understanding of 3D flows (cf. [21]).

In this paper, we consider the 2D inviscid heat conductive Boussinesq equations

$$\begin{cases} U_t + U \cdot \nabla U + \nabla P = \theta \mathbf{e}_2, \\ \theta_t + U \cdot \nabla \theta = \kappa \Delta \theta, \\ \nabla \cdot U = 0, \end{cases} \tag{1.1}$$

where $U = (u, v)$ is the velocity vector field, P is the scalar pressure, θ is the scalar temperature, the constant $\kappa > 0$ models thermal diffusion, and $\mathbf{e}_2 = (0,1)^{\mathrm{T}}$. In this paper, we consider (1.1) in a bounded domain $\Omega \subset \mathbb{R}^2$ with smooth boundary $\partial\Omega$. The system is supplemented by the following initial and boundary conditions:

$$\begin{cases} (U,\theta)(\mathbf{x},0) = (U_0,\theta_0)(\mathbf{x}), \quad \mathbf{x} \in \Omega, \\ U \cdot \mathbf{n}|_{\partial\Omega} = 0, \quad \theta|_{\partial\Omega} = \bar{\theta}, \end{cases} \tag{1.2}$$

where $\mathbf{n}$ is the unit outward normal to $\partial\Omega$, and $\bar{\theta}$ is a constant.

Received January 7, 2009. Revision received July 10, 2009.

The research of K. Zhao is partially supported by NSF Grant no. DMS0807406. The author would also like to thank his advisor, Prof. Ronghua Pan, for helpful discussion and suggestions on this paper.