Michigan Math. J. 59 (2010), 303–312

Effective Base Point Free Theorem for Log Canonical Pairs, II. Angehrn–Siu Type Theorems

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1. Introduction

The main purpose of this paper is to advertise the power of the new cohomological technique introduced in [Am]. By this new method, we generalize Angehrn–Siu type effective base point freeness and point separation (see [AS] and [Ko, 5.8 and 5.9]) for *log canonical* pairs. Here, we adopt Kollár's formulation in [Ko] because it is suitable for singular varieties. The main ingredients of our proof are the inversion of adjunction on log canonicity (see [Ka]) and the new cohomological technique (see [Am]). For the Kollár type effective freeness for log canonical pairs, see [F5]. In [F4], we give a simple new proof of the base point free theorem for log canonical pairs. It is closely related to the arguments in this paper.

The following theorems are the main theorems of this paper.

Theorem 1.1 (Effective Freeness; cf. [Ko, Thm. 5.8]). *Let (X, Δ) be a projective log canonical pair and M a line bundle on X. Assume that $M \equiv K_X + \Delta + N$, where N is an ample $\mathbb{Q}$-divisor on X. Let $x \in X$ be a closed point and assume that there are positive numbers $c(k)$ with the following properties.*

(1) *If $x \in Z \subset X$ is an irreducible (positive dimensional) subvariety, then*

$$(N^{\dim Z} \cdot Z) > c(\dim Z)^{\dim Z}.$$

(2) *The numbers $c(k)$ satisfy the inequality*

$$\sum_{k=1}^{\dim X} \frac{k}{c(k)} \le 1.$$

Then M has a global section not vanishing at x.

Theorem 1.2 (Effective Point Separation; cf. [Ko, Thm. 5.9]). *Let (X, Δ) be a projective log canonical pair and M a line bundle on X. Assume that $M \equiv K_X + \Delta + N$, where N is an ample $\mathbb{Q}$-divisor on X. Let $x_1, x_2 \in X$ be closed points and assume that there are positive numbers $c(k)$ with the following properties.*

Received December 2, 2008. Revision received July 21, 2009.