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An Explicit $\bar\partial$-Integration Formula for Weighted Homogeneous Varieties II. Forms of Higher Degree

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1. Introduction

Let Σ be a weighted homogeneous (singular) subvariety of $\mathbb{C}^n$. The main objective of this paper is to present a class of explicit integral formulas for solving the $\bar\partial$-equation $\omega = \bar\partial\lambda$ on the regular part of Σ, where ω is a $\bar\partial$-closed $(0,q)$-form with compact support and degree $q \geq 1$. Particular cases of these formulas yield L^p-bounded solution operators for $1 \leq p \leq \infty$ if Σ is a homogeneous and pure dimensional subvariety of $\mathbb{C}^n$ with an arbitrary singular locus.

As is well known, solving the $\bar\partial$-equation forms one of the main pillars of complex analysis; however, it also has deep consequences for algebraic geometry, partial differential equations, and other areas. For example, the classical Dolbeault theorem implies that the $\bar\partial$-equation can be solved in all degrees on a Stein manifold, and it is known that an open subset of $\mathbb{C}^n$ is Stein if and only if the $\bar\partial$-equation can be solved in all degrees (on that set). Nevertheless, it is usually not easy to produce an explicit operator for solving the $\bar\partial$-equation on a given Stein manifold, even if we know that it can be solved. The construction of explicit operators depends strongly on the geometry of the manifold on which the equation is considered. There exists a vast literature about this problem on smooth manifolds, both in books and papers (see e.g. [10; 11]).

The respective Dolbeault theory on singular varieties has been developed only recently. Let Σ be a singular subvariety of the space $\mathbb{C}^n$ and ω a bounded $\bar\partial$-closed differential form on the regular part of Σ. Fornæss, Gavosto, and Ruppenthal produced a general technique for solving the $\bar\partial$-equation $\omega = \bar\partial\lambda$ on the regular part of Σ, and they have successfully applied this technique to varieties defined by the formula $z^m = \prod_k w_k^{b_k}$ in $\mathbb{C}^n$; see [6; 9; 16]. Acosta, Solís, and Zeron have developed an alternative technique for solving the $\bar\partial$-equation (if ω is bounded) on the regular part of any singular quotient variety embedded in $\mathbb{C}^n$ that is generated by a finite group of unitary matrices—such as, for instance, hypersurfaces in $\mathbb{C}^3$ with only a rational double point singularity; see [1; 2; 21].

Nevertheless, the research on calculating explicit operators for solving the $\bar\partial$-equation $\omega = \bar\partial\lambda$ on the regular part of singular subvarieties $\Sigma \subset \mathbb{C}^n$ is still at a

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